

- CLAUDIO TERNULLO, *Intrinsic justification for large cardinals and structural reflection*.

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I argue that two main issues arise in connection with the justification of Large Cardinal Axioms: one is the ‘Intrinsicness Issue’, namely, the fact they do not seem to straightforwardly follow from the ‘concept of set’ (the ‘iterative concept of set’). The second is what I call the ‘Universality Issue’, the fact that there does not seem to exist any intrinsically motivated principle (or class of principles) equivalent to all known Large Cardinals. My strategy to tackle both issues is as follows. First, I examine the main ‘abstract motivating principles’ (in particular, Reflection, Resemblance and Uniformity) which have been introduced (and invoked) to justify large cardinals. As has been pointed out, such principles are not licensed by the concept of set only, insofar as they entail (or are best construed as entailing) the existence of classes. As a consequence, in the paper, I discuss several strategies to ‘stretch’ the concept of set so as to legitimise the use of (at least, some) class theory. Then, I proceed to review Bagaria’s Structural Reflection Principles which may turn out to be optimal in terms of justificatory strength, since they are motivated by several abstract principles (in particular, Resemblance and Uniformity), entail the use of a modest amount of class theory and may also be able to account for the existence of practically all known large cardinals.

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