

- STEVE AWODEY, *Homotopy type theory: Ten years after*.

Departments of Philosophy and Mathematics, Carnegie Mellon University, 5000 Forbes Ave., Pittsburgh, PA, 15213, USA.

E-mail: awodey@cmu.edu.

URL Address: <https://awodey.github.io>.

In the 10 years since the IAS Program on Univalent Foundations, which culminated in the release of the HoTT Book [9], substantial progress has been made in the field of homotopy type theory on several fronts, including solutions to leading open problems with both logical and mathematical significance. In work by Coquand et al. [3], the simplicial model of univalence [4] was shown have a constructive counterpart, verifying Voevodsky’s canonicity conjecture. A computational proof assistant [7] was engineered on this basis, and in 2022 was used to finally compute “Brunerie’s number” [5], finishing the formal verification of a proof that was begun at the IAS of the calculation of the fourth homotopy group of the 3-sphere, $\pi_4(S^3)$ [2].

The homotopical semantics of Martin-Löf type theory originated with [1], and was conjectured at the time by the author to provide an internal logic for higher toposes [6]. This was established by Shulman [8] in 2019, giving semantics for HoTT in all Grothendieck ∞ -toposes. This talk will report on current research relating the constructive models underlying the new generation of computational proof assistants with the classical homotopy theory of higher toposes.

[1] STEVE AWODEY AND MICHAEL WARREN, *Homotopy theoretic models of identity types*, **Mathematical Proceedings of the Cambridge Philosophical Society**, 2009.

[2] GUILLAUME BRUNERIE, *On the homotopy groups of spheres in homotopy type theory*, June 2016, (arXiv:1606.05916).

[3] CYRIL COHEN, THIERRY COQUAND, SIMON HUBER, AND ANDERS MÖRTBERG, *Cubical type theory: a constructive interpretation of the univalence axiom*, **21st International Conference on Types for Proofs and Programs**, May 2015, Tallinn, Estonia, pp. 129–162.

[4] CHRIS KAPULKIN AND PETER LEFANU LUMSDAINE, *The simplicial model of univalent foundations (after Voevodsky)*, **Journal of the European Mathematical Society**, 2021.

[5] AXEL LJUNGSTRÖM, *The Brunerie number is -2*, **Homotopy Type Theory (online)**, June 2022, homotopytypetheory.org/2022/06/09/the-brunerie-number-is-2.

[6] JACOB LURIE, *Higher topos theory*, **Princeton University Press**, 2009.

[7] ANDERS MÖRTBERG, *Cubical Agda*, **Homotopy Type Theory (online)**, December 2018, homotopytypetheory.org/2018/12/06/cubical-agda.

[8] MICHAEL SHULMAN, *All $(\infty, 1)$ -toposes have strict univalent universes*, April 2019, (arXiv:1904.07004).

[9] THE UNIVALENT FOUNDATIONS PROGRAM, *Homotopy type theory: the univalent foundations of mathematics*, The Institute for Advanced Study, 2013, homotopytypetheory.org/book.