

- GUILLERMO BADIA AND DAVID C. MAKINSON, *First-order friendliness*.

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The relation of *logical friendliness*, introduced in the propositional context in [1], has a very straightforward definition as a $\forall\exists$ version of the fundamental $\forall\forall$ notion of consequence. Specifically, if Γ is a set of formulae of classical propositional logic and ϕ is a formula of the same, Γ is said to be friendly to ϕ iff for every valuation v on the propositional variables occurring in formulae of Γ , if $v(\gamma) = 1$ for all $\gamma \in \Gamma$ then there is an extension of v to a valuation v' covering also any remaining variables in ϕ such that $v'(\phi) = 1$. It is thus a weakening of classical consequence and if the existential quantifier in its definition is replaced by a universal one, it reverts to the classical relation.

So defined, friendliness has a number of interesting features. While lacking some familiar properties of classical consequence, it satisfies some others in full, as well as yielding ‘local’ versions of yet others, as shown in [1]. However, if we seek to extend the definition from the propositional to the first-order context, a number of options arise due to the greater complexity of the notion of a first-order model, with its ingredients of domain of discourse, values for individual constants, values for predicate and function letters, and the equality relation. The various options generate distinct relations, which differ in their behaviour. Indeed, two of the lessons of the present paper are that the concept of friendliness is less robust in the first-order context than in the propositional one and that even the seemingly best behaved of the possible first-order options is less regular than its propositional counterpart, notable with respect to compactness, axiomatizability and interpolation.

The full paper is available on arXiv at <https://arxiv.org/abs/2210.13953>.

[1] D. Makinson. Friendliness and sympathy in logic. In: JY Beziau (ed.) *Logica Universalis* (2nd Edition), pp. 195-224, Basel: Birkhauser Verlag, 2007. (Essentially the same material appeared under the title “Friendliness for logicians” in *We Will Show Them! Essays in Honour of Dov Gabbay*, vol 2, ed S. Artemov et. al, College Publications 2005, pp 259-292).