▸ S. BONZIO, V. FANO, P. GRAZIANI AND M. PRA BALDI, *A logical modeling of severe ignorance via Bochvar external logic.*
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The study of ignorance is certainly as old as the study of knowledge; however the formal study of the logic of ignorance is still a young area of research. In the epistemological studies of ignorance the standard view is to define it as lack of knowledge (see for example [8], [5], [7], [6]). We believe that this is the reason why also the formal study of the logic of ignorance has been developed with reference to the formal study of the logic of knowledge. This tradition is mainly due to the work of Hintikka [4], who distinguishes two notions of lack of knowledge relative to an agent, namely "a (an agent) does not know that φ" ($\neg\mathbf{K}_a\varphi$) and "a does not know whether φ" ($\neg\mathbf{K}_a\varphi \wedge \neg\mathbf{K}_a\neg\varphi$). It seems that, according to Hintikka, only the latter explicates the notion of ignorance; indeed, he [4, p.12] formalizes ignorance (of an agent a) as $\neg\mathbf{K}_a\varphi \wedge \neg\mathbf{K}_a\neg\varphi$. Such regimentation has become standard in the logical literature on ignorance. For this reason, by the expression "ignorance as lack/absence of knowledge", we will refer to Hintikka's view throughout this talk.

However, in more recent times, van der Hoek and Lomuscio [11] introduced a modal logic (**Ig**) where ignorance is modeled by a primitive modal operator, unrelated to (lack of) knowledge. The spirit behind **Ig** is expressing "ignorance as a first class citizen" [11, p.3]. However, despite their intention, their solution does not seem too far from Hintikka's lack of knowledge. The semantics of **I**, as we will show, is the same as in Hintikka [4, p.12], with the only difference that **Ig** "can not speak" about knowledge. Similarly, the *Logic of Unknown Truths* (LUT) and the subsequent logics of ignorance proposed by Steinsvold [10] subordinate the concept of ignorance to that of knowledge. In these logics the black box (■) in fact stands for $\varphi \wedge \neg\mathbf{K}\varphi$; if the latter formula is true, and $\varphi \to \neg\mathbf{K}\neg\varphi$ holds, then also $\neg\mathbf{K}\varphi \wedge \neg\mathbf{K}\neg\varphi$ holds, which is again Hintikka's definition of ignorance.

Following the research trend opened in Fano and Graziani [2], this article intends to discuss the fact that *lack of knowledge* is just one way to look at ignorance and, taking up van der Hoek and Lomuscio's challenge, to introduce a logic which addresses the purpose of defining "ignorance as a first class citizen". In this paper, after discussing the consequences of defining ignorance as lack of knowledge (in the epistemic logic S_4), we introduce and investigate a modal logic having a primitive epistemic operator **I**, modeling ignorance. In particular, the idea we have in mind is that of modelling a type of *content-theoretic ignorance*, so to say an ignorance of something that stems from an unfamiliarity with its meaning, i.e. a *severe* notion of ignorance that implies a lack of awareness with respect to a subject-matter. In our view, this type of ignorance constantly affects the practice of science.

To achieve the goal of modeling severe ignorance, we base the semantics of our (modal) logic on the presence of a third truth-value, whose behaviour is infectious: we opt for Bochvar external logic, originally introduced in [1]. Our modal logic **SI** consists of a linguistic extension of Bochvar (external) logic via a primitive modality **I**, for ignorance.

The talk is organized into four parts: in the first part we introduce the standard (logical) approach to ignorance as "lack of knowledge". In the second part, we introduce Bochvar external logic. In particular, we extend the known results from [3], proving that Bochvar external logic is algebraizable with the quasivariety of Bochvar algebras as its equivalent algebraic semantics. In the third, it is introduced the logic **SI** of severe ignorance, its axiomatization for which we prove completeness with respect to a relational semantics and decidability (the proof follows the ideas of the modal logics based on weak Kleene logics introduced by Segerberg [9]). Finally, we conclude the talk with some remarks on the validity of certain formulas relevant to capture a severe notion of ignorance, and compare the differences between the standard view and the proposed logic for severe ignorance.

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