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Monadic NIP in monotone classes of relational structures.

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We prove that for any monotone class of finite relational structures, the first-order theory of the class is NIP in the sense of stability theory if, and only if, the collection of Gaifman graphs of structures in this class is nowhere dense. This generalises to relational structures a result previously known for graphs and answers an open question posed by Adler and Adler in [1].

The result is established by the application of Ramsey-theoretic techniques and shows that the property of being NIP is highly robust for monotone classes. We also show that the model-checking problem for first-order logic is intractable on any class of monotone structures that is not (monadically) NIP. This is a contribution towards the conjecture of Bonnet et al. from [2] that the hereditary classes of structures admitting fixed-parameter tractable model-checking are precisely those that are monadically NIP.

[1] HANS ADLER AND ISOLDE ADLER, Interpreting nowhere dense graph classes as a classical notion of model theory, European Journal of Combinatorics, vol. 36 (2014), pp. 322–330.

[2] BONNET, ÉDOUARD AND GIOCANTI, UGO AND DE MENDEZ, PATRICE OSSONA AND SIMON, PIERRE AND THOMASSÉ, STÉPHAN AND TORUŃCZYK, SZYMON, *Twin*width IV: ordered graphs and matrices, arXiv preprint, 2102.03117 (2021).