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Three-dimensional hypergraphical natural deduction.

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As is widely known, natural deduction was first presented in 1934, independently by Gentzen [2] and Jáśkowski [3]; this event gave rise to three fundamental and fundamentally different ways of rendering such deduction precise, a trio that firmly persists to the present day. Gentzen gave a tree format for natural deduction; Jáśkowski gave a box-based one, and a tabular, "bookkeeping" one. (Pelletier [4] credits Suppes with a fourth way, but this is controversial, since Suppes' innovation is a formalism for tracking suppositions that remain in force as a proof proceeds in Jáśkowskian tabular fashion.) We first briefly review the three natural-deduction ways, and show our expansion of Genzen's trees into a novel system based on (usually directed, acyclic) *hypergraphs*. (Hypergraphs are covered e.g. in [1] and — more recently — in [5].) We next show that our system (in two-dimensional mode) is implemented and integrated with automated reasoners (= "oracles"), via specimen formal proofs that range over third-order logic, with additional optional modal operators available for the alethic, epistemic, deontic cases etc. We then explain that the three original specifications for natural deduction, despite their differences, are most assuredly in any case two-dimensional: each element therein is located somewhere in a backdrop of an x and a y axis, as in simple, discrete Euclidean two-space. We then reveal how natural deduction in our hypergraphical environment can be better expressed in *three-dimensional* hypergraphs. Our 3D hypergraphical proofs use a third z axis on which formulae in nodes can be located, to and from which run inferential arcs. This third dimension, as we explain and show in relevant proofs, can be interpreted, within proof-theoretic semantics, as e.g. determining the degree of "prominence" of formulae and inferential links in a given proof.

We conclude with some remarks about connections we perceive between 3D hypergraphical natural reasoning and the dream of Leibniz to find a rigorous universal reasoning system. Leibniz dreamed of an interoperating pair: (i) the *calculus ratiocinator*, the machine or mechanical system, which brings information expressed in (ii) the universal rational calculus, or *characteristica universalis*, to life. Our system, we claim, realizes this dream.

[1] Claude Berge, ***Graphs and Hypergraphs***, North-Holland, 1973.

[2] Gerhard Gentzen, *Untersuchungen über das logische Schliezßen I & II*, ***Mathematische Zeitschrift***, vol. 39 (1934), pp. 176–210, 405–431.

[3] Stanislaw Jáśkowski, *On the Rules of Suppositions in Formal Logic*, ***Studia Logica***, vol. 1 (1934), no. 1, pp. 5–32.

[4] Francis Pelletier, *A Brief History of Natural Deduction*, ***History and Philosophy of Logic***, vol. 20 (1999), pp. 1–31.

[5] Vitaly Voloshin, ***Introduction to Graph and Hypergraph Theory***, Nova Kroshka, 2013.