▸ MARIA BEATRICE BUONAGUIDI, *Strong conditionals for paraconsistent arithmetics: comparisons in proof-theoretic strength.*
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The paraconsistent system of arithmetic developed by Weber [4] using the relevant logic subDLQ seems, on the face of it, to be deductively strong compared to other systems of paraconsistent arithmetic, such as Mortensen's [5]. Weber himself argues that the theory is devised explicitly for classical recapture, but provides no thorough proof-theoretic analysis of its strength. Another system of non-classical arithmetic showing interesting recapture results is HYPE arithmetic HYA, developed by Fischer et al. in [3]. In particular, Fischer et al. show that HYA is proof-theoretically equivalent to PA, and that, using the HYPE conditional, the standard lower bound proofs by Gentzen and Feferman-Schütte for transfinite induction in classical arithmetic and predicative analysis can be reproduced.

In this work, we compare subDLQ arithmetic with HYA proof-theoretically. There are several reasons why this comparison can be significant: indeed, both logics display paraconsistency and have a strong conditional satisfying the Deduction Theorem and Modus Ponens. However, while HYPE is sound and complete with respect to the class of involutive Routley frames [2], subDLQ is only nontrivial and does not have a class of models. We show that, while the strong conditional $\Rightarrow$ of subDLQ allows, similarly to HYPE's $\rightarrow$, to reproduce the Gentzen lower bound proof for transfinite induction in classical arithmetic, the "amount" of paraconsistency we observe in subDLQ does not allow the proof to carry through for all formulae of the full language of arithmetic. We obtain a syntactically definable class Ψ of well-behaved sentences in subDLQ arithmetic for which the proof carries through, but $\Psi \subset \mathtt{Sent}_{\mathcal{L}_{\mathtt{subDLQ}}}$, due to the non-classical behaviour of identity. Conversely, while in HYA, like in subDLQ arithmetic, identity is defined as an equivalence relation, the behaviour of paraconsistency and paracompleteness in HYPE forces identity to behave classically, allowing the proof to go through.

[1] Hannes Leitgeb, *HYPE: a system of hyperintensional logic*, ***Journal of philosophical logic***, vol. 48 (2019), no. 2, pp. 305–405.

[2] Stanislav O. Speranski, *Negation as a modality in a quantified setting*, ***Journal of Logic and Computation***, vol. 31 (2021), no. 5, pp. 1330–1355.

[3] Martin Fischer, Carlo Nicolai and Pablo Dopico, *Nonclassical truth with classical strength. A proof-theoretic analysis of compositional truth over HYPE*, ***The Review of Symbolic Logic***, (2021), pp. 1–24.

[4] Zach Weber, ***Paradoxes and inconsistent mathematics***, Cambridge University Press, 2021.

[5] Chris E. Mortensen, ***Inconsistent mathematics***, Springer Science & Business Media, 2013.