▶ ALAKH DHRUV CHOPRA, FEDOR PAKHOMOV, *Finite labeled trees ordered by non-inf-preserving embeddings.*

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We study the well-quasi-order (wqo) consisting of the set of finite trees with internal and leaf labels coming from arbitrary wqo's P and Q respectively, ordered by homomorphic embeddability which respects the order of the labels. This is a variant of the usual Kruskal ordering but without infima preservation. We calculate the precise maximal order types of these wqo's — in the style of De Jongh, Parikh, and Schmidt — as a function of the maximal order types of P and Q. In doing so, we sharpen some upcoming results of Andreas Weiermann and Harvey Friedman. This also helps to calibrate the reverse mathematical strength of certain well-foundedness assertions and obtain natural combinatorial independence results.

Nash-Williams proved that arbitrary transfinite sequences using finitely many elements from a well-quasi-ordered set are also well-quasi-ordered, but the proof does not offer immediate information about the maximal order type. Erdos and Rado previously proved this for the specific case of sequences of length ω^n using a more concrete approach. Our results lead to precise bounds for transfinite sequences of length less than ω^ω, using the correspondence between the set of labeled finite trees and indecomposable transfinite sequences of finite range with length less than ω^ω. Specific instances of this wqo have also been considered by Marcone and Montalban in their study of a limited form of Fraisse's Conjecture.

[1] D.H.J DE JONGH, ROHIT PARIKH, *Well-partial orderings and hierarchies*, **Indagationes Mathematicae (Proceedings)**, vol. 80, issue 3, 1977, pp. 195–207.

[2] DIANA SCHMIDT, *Well-Partial Orderings and their Maximal Order Types*, **Well-Quasi Orders in Computation, Logic, Language and Reasoning**, (Peter M. Schuster, Monika Seisenberger, Andreas Weiermann, editors), Springer Cham., 2020, pp. 351–391.

[3] HARVEY M. FRIEDMAN, ANDREAS WEIERMANN, *Some independence results related to finite trees*, **Philosophical Transactions of the Royal Society A**, vol. 381, issue 2248.