

- GRETA CORAGLIA, SHREYA ARYA, ANA LUIZA DA CONCEIÇÃO TENORIO, PAIGE NORTH, SEAN O’CONNOR, AND HANS RIESS, *Categorical methods for fuzzy type theory*.

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We consider classical models for dependent types à la Martin-Löf, such as display map categories [1, 2], and we revisit them in the context of enriched category theory to obtain a fuzzy version of traditional deductive systems. The long-distance scope of this project is to recover many results in opinion dynamics, precisely in the context of cellular sheaves [3], in a fuzzy environment.

Our aim is to – very roughly – model opinions, so we first start by saying what it is that we consider to be an opinion. On this, we follow the path suggested by the correspondence that is mostly known as Curry-Howard: since there is a one-to-one correspondence between logics and programming languages, which kind of looks like the following

proofs	executions (terms)
formula	program (type)

what we do is simply add a leg to it:

proofs	executions (terms)	motivations
formula	program (type)	belief

In our setting, programs are beliefs and executions are thoughts that lead to holding such beliefs. This is of course reductive of the human mind. Nevertheless, we might be able to gain some insight out of it.

Now, given a belief and a motivation for it, someone might consider it good or bad. For example, they could motivate the belief “bees should be protected” either with “they carry pollen between plants” or “I like honey”, but perhaps these are not influential to the same extent. This is where fuzziness comes into play.

We present here the first step in the process: we introduce a theory of fuzzy types with their structural rules, and prove soundness and completeness for their calculus. We begin the analysis of possible extension to connectives and type constructors.

This work is part of the Adjoint School project.

[1] PAUL TAYLOR, *Practical Foundations of Mathematics*, Cambridge Studies in Advanced Mathematics, Cambridge University Press, 1999.

[2] MARTIN HYLAND, ANDREW PITTS, *The theory of constructions: categorical semantics and topos-theoretic models*, *Contemporary mathematics, American Mathematical Society*, vol. 2 (1989), pp. 137–199.

[3] ROBERT GHRIST, HANS RIESS, *Cellular sheaves of lattices and the Tarski Laplacian*, *Homology, Homotopy and Applications, International Press of Boston*, vol. 24 (2022), no. 1, pp. 325–345.