▸ FRANCESCO DAGNINO, *Quotients in Relational Doctrines.*
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Among categorical approaches to logic, Lawvere's doctrines [2, 3] stand out as a simple and powerful framework capable to cope with a large variety of situations. Doctrines are functors $P : \mathcal{C}^{\mathrm{op}} \to \mathcal{P}os$, on a category $\mathcal{C}$ with finite products, providing an algebraic description of theories in predicate logic: objects and arrows of $\mathcal{C}$ model contexts and terms, products of $\mathcal{C}$ model context concatenation and P maps each object to a poset that models formulas on that object, ordered by logical entailment. Doctrines have proved to be very effective in studying quotients as well as universal constructions freely adding them to any doctrine [4, 5], so that one can use quotients even though they are not natively available.

A longstanding variable-free alternative to predicate logic is the *calculus of relations* [1, 6, 7]. Here one takes as primitive concepts (binary) relations instead of (unary) predicates, with some basic operations, such as relational identities, composition and converse. However natural, this calculus has been much less studied using functorial tools. In this talk we introduce *relational doctrines* as a functorial description of the calculus of relations. We show that relational doctrines are a natural setting where to deal with quotients. Then, we formulate a universal construction that freely adds quotients to any relational doctrine, generalising the elementary quotient completion of an existential elementry doctrine [4, 5]. Moreover, thanks to the variable-free nature of the calculus, we can get rid of products in the base category, thus recovering many new instances, such as, the exact completion of a category with weak finite limits and categories of metric structures, like metric spaces and non-expansive maps.

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