▸ DAVID ELLERMAN, *Partition logic and its applications.*
Faculty of Social Sciences, University of Ljubljana, Ljubljana 1000, Slovenia.
E-mail: david@ellerman.org.

Propositional logic is a special case of the Boolean logic of subsets (when the universe set is $U = 1$). Subsets and partitions (or quotient sets) are category-theoretic duals so there is a dual logic of partitions. The join and meet operations on partitions were defined in the 19th century (Dedekind and Schr̈oder), but the definition of the implication operation on partitions and two algorithms for defining all the Boolean operations on partition only came in the 21st century [1]. This development of the logic of partitions led to a series of applications.

Just as Boole's first application was the quantitative version of the logic of subsets in finite probability theory, so the first application of partition logic was the quantitative version as the logical information theory based on the notion of logical entropy [2]. The logical entropy of a partition is a probability measure, the probability in two random draws from U that one will draw a distinction of the partition (i.e., a pair of elements in different blocks of the partition). All the usual Venn diagram definitions for simple, joint, conditional, and mutual logical entropy follow from it being a measure. The better-known Shannon entropy is not a measure on a set but there is a non-linear monotonic transformation of those compound logical entropy formulas that yields all the corresponding formulas for Shannon entropy. Thus logical entropy provides a new logical foundation for information theory which includes the Shannon entropy as a specialized formula that is central to coding and communications theory. Logical entropy also generalizes to the quantum level where it is the probability that in two independent measurement of the same quantum state, two different eigenvalues will be obtained.

The second application of partition logic is to the century-old problem of interpreting quantum mechanics (QM). When the set-based concepts of partition mathematics are linearized (a standard method in the mathematics folklore) to vector spaces, e.g., Hilbert spaces, then one arrives at the mathematical formalism of QM [3]–not the physics of QM that must be brought in from classical physics by quantization. Since partitions are the mathematical tool to describe distinctions and indistinctions or distinguishability and indistinguishability, this shows that QM math is the mathematics of (objective) indefiniteness and definiteness. One can even see the lattice of partitions on a set as the bare bones or skeletal version (i.e., stripping away the scalars) of the pure, mixed, and classical states in QM. Moreover, the other basic QM concept is that of an observable operator where the direct-sum decomposition of its eigenspaces is just the linearized version of the inverse-image partition of a numerical attribute $f : U \to \mathbb{R}$ Thus both the quantum states and observables are derived from partitions and the basic notion of measurement, measuring a state by an observable, is represented back at the set level by the join of the two partitions. This way of interpreting the quantum formalism is the "partitional interpretation" of quantum mechanics.

[1] David Ellerman, *The Logic of Partitions: Introduction to the Dual of the Logic of Subsets*, ***Review of Symbolic Logic***, vol.3, no.2, pp.287 - 350.

[2] David Ellerman, ***New Foundations for Information Theory: Logical Entropy and Shannon Entropy***, SpringerNature, 2021.

[3] ——— *Follow the Math!: The Mathematics of Quantum Mechanics as the Mathematics of Set Partitions Linearized to (Hilbert) Vector Spaces*, ***Foundation of Physics***, vol.52, no.5, 100 (2022).