▸ JACOPO EMMENEGGER, FABIO PASQUALI, AND GIUSEPPE ROSOLINI, *Quotients and equality, (co)algebraically, and the elimination of imaginary elements.*
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Doctrines were introduced by Lawvere [2, 3] as an algebraic tool to work with logical theories and their extensions. A doctrine consists of a poset indexed on a category, and the prime example is the Lindenbaum-Tarski algebra of well-formed formulas in context, indexed on the category of contexts (*i.e.* lists of sorts) and substitutions (*i.e.* lists of terms).

The algebraic character of the theory of doctrines makes it a suitable context where to address the question: "What is the theory obtained by (co)freely adding logical structure?" or the closely related question: "How to express additional logical structure in terms of what is already available?". Technically speaking, we look at the 2-category of doctrines and its subcategories on doctrines (read theories) with conjunctions, or with equality, or with all connectives and quantifiers from first order logic, and then ask whether whether a certain "forgetful" functor is adjoint and, in the second case, whether the adjunction obtained in this way is (co)monadic.

After an introduction to doctrines and their connection to logic and type theory, I shall present the main results of [1] discussing the above questions in the case of two forgetful functors: the one from theories with conjunctions, equality and quotients to theories with conjunctions and equality, and the one that further forgets equality. Not surprisingly, the answers revolve around the concept of equivalence relation. I shall discuss applications to useful constructions in categorical logic and type theory, as well as to the elimination of imaginary elements in the sense of Poizat [4]. If time allows, I shall also describe how to lift this setting to Grothendieck fibrations (of which doctrines are a particular case) using groupoids instead of equivalence relations.

[1] J. Emmenegger, F. Pasquali, and G. Rosolini. Elementary doctrines as coalgebras. J. Pure Appl. Algebra, (224), 2020.

[2] F. W. Lawvere. Adjointness in foundations. Dialectica, 23:281–296, 1969.

[3] F.W. Lawvere. Equality in hyperdoctrines and comprehension schema as an adjoint functor. In A. Heller, editor, Proc. New York Symposium on Application of Categorical Algebra, pages 1–14. Amer.Math.Soc., 1970.

[4] B. Poizat. Une théorie de Galois imaginaire. J. Symbolic Logic, 48(4):1151–1170 (1984), 1983.