A Family of Contrastructural Classical Logics

[Redacted]

The standard presentation of multiple-conclusion classical logic stipulates that an argument is valid just in case whenever *all* the premises are true *some* conclusion is true. Thus, it imposes what we call an *existential* reading of conclusions and a *universal* reading of premises. This is the orthodox reading of multiple-conclusion consequence in philosophical logic, but it is not the only one possible. In this talk, we introduce and study a family of logical systems, each of which results by taking multiple-conclusion classical logic and modifying the definition of validity so as to induce a specific (often heterodox) reading of premises and/or conclusions. More precisely, the family consists of the 16 systems that can be defined by means of the following schema:

$$\Gamma \models^{Q_1 Q_2} \Delta \quad \text{iff} \quad \forall v \in \mathcal{V} : \quad \text{if } (Q_1\, \gamma \in \Gamma)\ v(\gamma) = 1 \text{ then } (Q_2\, \delta \in \Delta)\ v(\delta) = 1$$

where Γ and Δ are sets of sentences, $\mathcal{V}$ is the set of classical interpretations of the language, and Q_1 and Q_2 are any quantifiers in the set $\{\forall, \exists, \hat{\forall}, \hat{\exists}\}$, with $\hat{\forall}$ and $\hat{\exists}$ defined thus:

$$\begin{aligned} (\hat{\forall}\ \sigma \in \Sigma)\phi &:= \Sigma \neq \varnothing \wedge (\forall \sigma \in \Sigma)\phi \\ (\hat{\exists}\ \sigma \in \Sigma)\phi &:= \Sigma = \varnothing \vee (\exists \sigma \in \Sigma)\phi \end{aligned}$$

(From an intuitive standpoint, $\hat{\forall}$ and $\hat{\exists}$ can be understood as restricted quantifiers with and without existential import on their domain of quantification, respectively.) When Q_1 is $\forall$ and Q_2 is $\exists$, the above schema delivers standard multiple-conclusion classical logic, here denoted $\mathbf{CL}^{\forall\exists}$. For all other cases, the resulting systems deviate from $\mathbf{CL}^{\forall\exists}$ both in their valid inferences and in their structural properties; interestingly, they not only lack some structural properties that classical logic enjoys but also enjoy some structural properties that classical logic lacks. This is why we call them *contrastructural classical logics.*

The talk has three main parts. In the first part, we study our logics from a model-theoretic standpoint; we show how they are ordered by inclusion and analyze their structural properties. In the second part, we study our logics from a proof-theoretical standpoint; we provide a recipe for constructing a sound and complete sequent calculus for each of them. Lastly, in the third part, we discuss the informal interpretation and potential applications of the systems presented; we argue that they can be viewed as allowing the application of classical reasoning to different epistemic contexts (where the agent has particular informational resources and goals).