

- RENÉ GAZZARI, *Defining Natural Deduction derivations using a formal theory of occurrences*.

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Distinctive feature of Gentzen's [2] Natural Deduction is the possibility of *discharging* previously assumed statements. A well-defined representation of such a facultative discharge (as proposed by Gentzen) requires the notion of *occurrences* of assumptions and inference steps in derivations. Even though Gentzen and his successors, in particular Prawitz [3], were aware of the necessity to deal with occurrences (instead of considering only the syntactic entities themselves), little to nothing more than an intuitive account is found in the literature. Only recently, Gazzari [1] developed a full-fledged formal theory of occurrences capable of representing occurrences in a generality needed for adequately defining the discharge function and, this way, derivations.

We provide relevant concepts of the theory of occurrences and, based on these concepts, central proof-theoretic definitions, as of assumptions, inference steps, discharge functions and, in the end, of derivations. This is a foundational topic, of conceptual character, whose purpose is to give insight into the underlying technical complexity of the usual intuitive treatment of derivations and the assurance that a detailed formal definition of this intuitive treatment can be obtained.

[1] RENÉ GAZZARI, *Formal Theories of Occurrences and Substitutions*, Dissertation, University of Tübingen, 2020.

[2] GERHARD GENTZEN, *Untersuchungen über das logische Schließen*, *Mathematische Zeitschrift*, vol. 39 (1935), pp. 176–210.

[3] DAG PRAWITZ, *Natural Deduction: A Proof-Theoretical Study*, Almqvist & Wiksell, 1956.