▸ VALENTIN GORANKO, *On modal logics with weakly transitive accessibility relations.*
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Say that binary relation R on a set W has the *weak transitivity property* if for every $u, v \in W$, if v is reachable from u in 3 R-steps, then v is reachable from u in 2 R-steps. Formally, weak transitivity is expressed by the first order sentence

$$\forall u \forall v(\exists x \exists y(uRx \wedge xRy \wedge yRv) \rightarrow \exists z(uRz \wedge zRv)).$$

Clearly, every transitive relation is weakly transitive, but not vice versa. Non-transitive but weakly transitive relations do not naturally occur often, but there are several interesting and diverse cases where they do, including:

- The edge relation in the countable random graph (aka, Rado graph); more generally, in any graph of diameter 2.
- The comparability relation between nodes in forests, regarded as models of branching time, where that relation can be naturally defined as "*being on the same history (timeline)*".
- The right (or left) neighbourhood relation between two intervals on a linear order, where an interval j is a right neighbour of the interval i iff the end of i and the beginning of j coincide.

Weak transitivity is frame-definable by the modal formula $\Diamond\Diamond\Diamond p \rightarrow \Diamond\Diamond p$ or, equivalently, by $\Box\Box p \rightarrow \Box\Box\Box p$. Added as an axiom to the modal logic $\mathbf{K}$ it defines the simplest normal modal logic, $\mathbf{K}_3^2$, for which, to my knowledge, no published works yet have proved or disproved decidability, nor finite model property.

This work presents and discusses some modal logics (including those associated with the cases above) containing $\mathbf{K}_3^2$ but not $\mathbf{K}_4$ (i.e., with weakly transitive but generally non-transitive relations), including some known and some new results about representation theorems, finite model property, and decidability for them.