▸ PIOTR GRUZA, *The structure of the definability relation between definitions of truth*.
Institute of Mathematics, University of Warsaw, S. Banacha 2, 02-097 Warsaw, Poland.
E-mail: p.gruza3@uw.edu.pl.

A theory T is a *theory of truth* for a language $\mathcal{L}$ if and only if there exists a formula Θ such that for each sentence $\sigma \in \mathcal{L}$, the theory T proves $\Theta(\ulcorner\sigma\urcorner) \leftrightarrow \sigma$. The well-known theorem of Alfred Tarski states that a reasonably strong theory cannot be a theory of truth for its language.

In this talk, we focus on finitely axiomatizable theories of truth (called *definitions of truth*) for the language of Peano Arithmetic extending $\mathrm{I}\Delta_0 + \mathrm{Exp}$. For simplicity, we assume that all considered theories are expressed in languages extending $\mathcal{L}_{\mathrm{PA}}$ by relational symbols only.

Having two theories of truth S and T, we say that S *defines* T iff we can assign to every non-arithmetic n-ary symbol $\mathfrak{R}$ of $\mathcal{L}_T$ an n-ary formula $\Theta_{\mathfrak{R}}$ of $\mathcal{L}_S$ in such a way that S proves every axiom of T with $\Theta_{\mathfrak{R}}$ substituted for each occurrence of $\mathfrak{R}$ for each symbol $\mathfrak{R}$ – in other words, S directly and conservatively over $\mathcal{L}_{\mathrm{PA}}$ interprets T. It can be seen that a definability relation constitutes a preorder on the theories of truth ($S \geq T$ iff S defines T).

Using a method developed by Fedor Pakhomov and himself, Albert Visser showed that there is no minimal element in the definability preorder among definitions of truth. Combining that method with some truth-theoretic techniques, we prove that the order generated by that preorder is a distributive lattice which embeds every countable distributive lattice.

Joint work with Mateusz Łełyk.