

- ISTVÁN JUHÁSZ, *Resolvability of product spaces and measurable cardinals.*

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All spaces below are crowded (i.e. have no isolated points). A space is  $\kappa$ -*resolvable*, if it can be partitioned into  $\kappa$  disjoint dense subsets. It is *irresolvable* if it is not 2-resolvable.  $X$  is *maximally resolvable* if it is  $\Delta(X)$ -resolvable, where  $\Delta(X)$  is the minimum size of a non-empty open set in  $X$ . It is easy to see that the product of infinitely many spaces is always  $\mathfrak{c}$ -resolvable.

For  $n \leq \omega$  let  $M(n)$  be the statement that there are  $n$  measurable cardinals and  $\Pi(n)$  ( $\Pi^+(n)$ ) that there are  $n + 1$  (0-dimensional  $T_2$ ) spaces whose product is irresolvable. We prove that

1.  $M(1)$ ,  $\Pi(1)$  and  $\Pi^+(1)$  are equiconsistent;
2. if  $1 < n < \omega$  then  $CON(M(n))$  implies  $CON(\Pi^+(n))$ ;
3.  $CON(M(\omega))$  implies the consistency of having infinitely many 0-dimensional  $T_2$ -spaces such that the product of any finitely many of them is irresolvable.

These results settle old problems of Malykhin.

An even older question of Ceder and Pearson asks if the product of a maximally resolvable space with any space is itself maximally resolvable. Concerning this we show that the following are consistent modulo  $M(1)$ :

1. There is a 0-dimensional  $T_2$  space  $X$  with  $\omega_2 \leq |X| = \Delta(X) \leq 2^{\omega_1}$  whose product with any countable space is not  $\omega_2$ -resolvable.
2. There is a monotonically normal space  $X$  with  $|X| = \Delta(X) = \aleph_\omega$  whose product with any countable space is not  $\omega_1$ -resolvable.

These significantly improve a result of Eckertson. Unlike in the first part, here we do not know if large cardinals, or even anything more than ZFC, are needed to get a counterexample to the Ceder-Pearson question.

This is joint work with L. Soukup and Z. Szentmiklóssy.

[1] I. JUHÁSZ, L. SOUKUP, AND Z. SZENTMIKLÓSSY, *On the resolvability of products*, *Fundamenta Mathematicae*, vol. 260 (2023), pp. 281–295.