▸ EITETSU KEN, *On Σ_0^B-generalizations of counting principles over V^0.*
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Ajtai's discovery ([1]) of $V^0 \nvdash ontoPHP_n^{n+1}$, where $ontoPHP_n^{n+1}$ is a Σ_0^B formalization of the statement "there does not exist a bijection between $(n+1)$ pigeons and n holes," was a significant breakthrough in proof complexity, and there have been many interesting generalizations and variations of this result.

In this talk, we first focus on the following well-known result ([2]): for any $p \geq 2$,

$$V^0 + Count_k^p \nvdash injPHP_n^{n+1},$$

where $Count_k^p$ denotes a Σ_0^B formalization of the modular counting principle mod p and $injPHP_n^{n+1}$ denotes that of the pigeonhole principle for injections.

We try to make this result uniform for p. We give three types of (first-order and propositional) formulae which at first glance seem to be generalized versions of counting principles, and compare their strengths over V^0. In particular, we see two of them, $UCP_n^{l,d}$ and GCP, actually serve as uniform versions of $Count_n^p$ $(p \geq 2)$.

Then we conjecture that $V^0 + UCP_k^{l,d} \nvdash injPHP_n^{n+1}$ and give a sufficient condition to prove it.

[1] Ajtai, M., *The complexity of the Pigeonhole Principle*, ***Combinatorica***, vol.14 (1994), no. X, pp.417–433.

[2] Beame, P., & Riis, S., *More on the relative strength of counting principles*, ***Proof Complexity and Feasible Arithmetics*** (P. Beame, & S. Buss), American Mathematical Society, Providence, RI, 1998, pp.13–35.