▶ BEIBUT KULPESHOV, SERGEY SUDOPLATOV, *Almost quite orthogonality of 1-types in weakly o-minimal theories.*

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The present lecture concerns the notion of *weak o-minimality* originally studied by H.D. Macpherson, D. Marker and C. Steinhorn in [1]. A *weakly o-minimal structure* is a linearly ordered structure $M = \langle M, =, <, \dots \rangle$ such that any definable (with parameters) subset of M is a finite union of convex sets in M.

Here we study a new variant of orthogonality of non-algebraic 1-types in weakly o-minimal theories: almost quite orthogonality.

We need the notion of a (p, q)-splitting formula introduced in [2]. Let $A \subseteq M$, $p, q \in S_1(A)$ be non-algebraic, $p \not\perp^w q$. We say that an L_A-formula $\phi(x, y)$ is a *(p, q)-splitting formula* if there exists $a \in p(M)$ such that $\phi(a, M) \cap q(M) \neq \emptyset$, $\neg\phi(a, M) \cap q(M) \neq \emptyset$, $\phi(a, M) \cap q(M)$ is convex and $\inf[\phi(a, M) \cap q(M)] = \inf q(M)$.

Let T be a weakly o-minimal theory, $M \models T$, $A \subseteq M$, $p, q \in S_1(A)$ be non-algebraic. We say that p is not *almost quite orthogonal* to q if there exist a (p, q)-splitting formula $\phi(x, y)$ and an A-definable equivalence relation $E_q(x, y)$ partitioning $q(M)$ into infinitely many convex classes so that for any $a \in p(M)$ there is $b \in q(M)$ such that $\sup \phi(a, M) = \sup E_q(b, M)$. We say that T is *almost quite o-minimal* if the notions of weak and almost quite orthogonality of 1-types coincide.

THEOREM 1. *Let T be a weakly o-minimal theory of finite convexity rank having less than 2^ω countable models, $\Gamma_1 = \{p_1, p_2, \dots, p_m\}$, $\Gamma_2 = \{q_1, q_2, \dots, q_l\}$ be maximal pairwise weakly orthogonal families of quasirational and irrational 1-types over $\emptyset$ respectively for some $m, l < \omega$. Then T has exactly $3^m 6^l$ countable models iff T is almost quite o-minimal.*

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[1] H.D. MACPHERSON, D. MARKER, AND C. STEINHORN, *Weakly o-minimal structures and real closed fields*, ***Transactions of the American Mathematical Society***, Vol. 352, No. 12 (2000), pp. 5435–5483.

[2] B.SH. KULPESHOV, *Criterion for binarity of $\aleph_0$-categorical weakly o-minimal theories*, ***Annals of Pure and Applied Logic***, Vol. 45 (2007), pp. 354–367.