▸ MATEUSZ ŁEŁYK, *On sets definable via pathologies in satisfaction classes.*
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A *satisfaction class* in a nonstandard model $\mathcal{M}$ of Peano Arithmetic (PA) is essentially a satisfaction relation which validates the usual compositional Tarski's conditions for all formulae *in the sense of* $\mathcal{M}$. While a classical theorem, originally due to Kotlarski, Krajewski and Lachlan, shows that every countable and recursively saturated model of PA carries a satisfaction class, recent developments provided many limitative results on how well-behaved can such a satisfaction class be. In particular, it was shown in [1], that unless $\mathcal{M}$ satisfies a proper and rather strong extension of PA (all finite iterations of uniform reflection over PA), every satisfaction class S on $\mathcal{M}$ exhibits the following pathology: there is a disjunction with a nonstandard number of disjuncts which is true (according to S) but whose each disjunct is deemed false (by S). Somewhat surprisingly, in an arbitrary countable and recursively saturated model $\mathcal{M}$ of PA one can find a satisfaction class S which behaves correctly on arbitrarily long disjunctions in the following sense: every disjunction with a true (according to S) disjunct is itself true (according to S).

In the talk we scrutinize this picture and study various variants of the following general question: when for a subset X of a countable recursively saturated model $\mathcal{M}$ of PA one can find a satisfaction class S such that X is the set of lengths of those disjunctions on which S behaves correctly? We solve this problem completely for the special case of idempotent disjunctions, i.e. the ones in which every disjunct is the same formula (but repeated many times). Firstly, we show that for a set $X \subseteq \mathcal{M}$, satisfying some minimal obvious conditions, there is a satisfaction class S such that X is precisely the set of lengths of those idempotent disjunctions of the sentence $0 = 1$ which are false (according to S) if and only if X satisfies a weak saturation condition we call *separability*. As a consequence, we show that in a model $\mathcal{M}$ the standard cut ω can be defined via a satisfaction class in this way if and only if $\mathcal{M}$ is arithmetically saturated. Secondly, we show when for a cut $I \subseteq \mathcal{M}$ there is a satisfaction class S such that I consists of those numbers c such that S behaves correctly on *every* idempotent disjunction of length $\leq c$. We show that in a (countable and recursively saturated) model $\mathcal{M}$ every cut can be characterised in this way if and only if $\mathcal{M}$ is arithmetically saturated. Finally we show how to generalize these results to various other propositional operations like taking idempotent conjunctions and adding long blocks of double negations.

This is joint work with Athar Abdul-Quader which builds upon an unpublished work due to James Schmerl.

[1] Cezary Cieśliński, Mateusz Łełyk, Bartosz Wcisło, *The Two Halves of Disjunctive Correctness*, ***Journal of Mathematical Logic***, https://doi.org/10.1142/S021906132250026X