

- STEPHEN MACKERETH AND JEREMY AVIGAD, *Two-sorted Frege Arithmetic is not conservative.*

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Neo-Fregean logicians Hale and Wright [1] have claimed that Hume's Principle (HP) may be taken as an implicit, stipulative definition of cardinal number, true simply by fiat. A longstanding problem for neo-Fregean logicism is that HP is not deductively conservative over the theory to which it is added, namely, pure axiomatic second-order logic. This seems to preclude HP from being true by fiat. In this talk, we study Richard Kimberly Heck's [2] theory of Two-sorted Frege Arithmetic (2FA), a variation on HP which has been thought to be deductively conservative over second-order logic. We show that it isn't. In fact, 2FA is not conservative over  $n$ -th order logic, for all  $n \geq 2$ . It follows that in the usual one-sorted setting, HP is not deductively Field-conservative (in the sense of Weir [3]) over second- or higher-order logic.

[1] BOB HALE AND CRISPIN WRIGHT, *The Reason's Proper Study: Essays towards a Neo-Fregean Philosophy of Mathematics*, Clarendon Press, Oxford, 2001.

[2] RICHARD KIMBERLY HECK, *The Julius Caesar objection*, *Language, Thought, and Logic: Essays in Honour of Michael Dummett* (Richard Kimberly Heck, editor), Oxford University Press, Oxford, 1997, pp. 273–308.

[3] ALAN WEIR, *Neo-Fregeanism: An embarrassment of riches*, *Notre Dame Journal of Formal Logic*, vol. 44 (2003), no. 1, pp. 13–48.