

- ROBIN MARTINOT AND FRANCESCA POGGIOLESI, *Purity and explanatoriness of proof*.

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Philosophy of mathematics distinguishes several qualities or ideals of proof, and in this talk we will focus on two such ideals. First, ‘purity’ of proof restricts the methods of a proof to those that in some sense intrinsically belong to a theorem. Here, notions that are thought extraneous to a theorem are excluded from a proof. Second, ‘explanatoriness’ of proof is the property that a proof not only convinces us that a theorem is true, but additionally shows us *why* it holds.

We aim to investigate the interaction between purity and explanatoriness. From [2] and [4], we gather that Bolzano considered purity and explanatoriness to go together. It seems natural that purity provides an explanation by providing the true reasons that a theorem is true, while impurity raises the question where the extraneousness comes from. On the other hand, a specific kind of explanation is also thought to go together with impurity of proof: impurity unifies results and shows previously unnoticed connections, and “in such a case, the more general context has greater explanatory power” [3]. Additionally, there are suggestions that “a proof’s explanatory power is independent of its purity” [5].

We will study the relation between purity and explanatoriness further by considering particular notions of purity (e.g., ‘topical’ purity [1]) and explanatoriness (e.g., ‘conceptual’ explanatoriness [7]), and analyzing their behaviour for various proofs of theorems such as the Infinitude of Primes and Pythagoras’ Theorem. A broader analysis of the behaviour of these notions will give us more insight into the ideals of purity and explanation themselves, and the differences between such ideals.

Finally, there has been some research on purity and explanatoriness in the context of formal (syntactic) derivations (see e.g. [6]). We will conclude by giving an expectation of the interaction of these properties in the formal setting.

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