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Polarization of light is a quantum phenomenon which is directly accessible to our senses. It also gives rise to the three-polarizer paradox in quantum mechanics.

We present a constructive logic solution to this paradox, inspired from Curry-Howard correspondence [3]. This quantum constructive logic differs from quantum logics in the style of Birkhoff-von Neumann [1].

Formulae are defined by the following formal grammar, where $\sqcup$ is a non-Boolean formula constructor corresponding to quantum physical state superposition, F stands for falsehood, and elements of $\mathcal{B}$ are distinct

$$\begin{array}{ll} \mathcal{F} ::= \mathcal{F}_c \mid \mathcal{Q} & \mathcal{F}_c ::= \mathsf{F} \mid \mathcal{E} \mid (\mathcal{E} \to \mathsf{F}) \\ \mathcal{Q} ::= \sqcup \mathcal{B} & \mathcal{B} ::= \{\mathcal{E}, \mathcal{E}\} \\ \mathcal{E} ::= \sigma_1 \mid \sigma_2 \mid \ldots & \end{array}$$

The domain of proof terms decomposes into three categories: set $\boldsymbol{T}$ of *quantum transition terms* (or τ-*terms*), $\boldsymbol{A}$ of *amplitude terms* and $\boldsymbol{B}$ of *base terms*, defined by the following formal grammar

$$\begin{aligned} \boldsymbol{T} &::= (\!|\boldsymbol{A}, \boldsymbol{A}|\!) \mid \widehat{\delta}_{\mathbf{I}}\boldsymbol{A} \mid \boldsymbol{T}^{\mathbf{W}} \mid (:\boldsymbol{T}/\boldsymbol{T}) \mid (:\widehat{\delta}_{\mathbf{I}}/\boldsymbol{T}) \mid \boldsymbol{\omega}\left(\frac{\boldsymbol{T}}{\boldsymbol{T}}\right) \\ \boldsymbol{A} &::= \mathbf{0} \mid \boldsymbol{B} \\ \boldsymbol{B} &::= V \mid \boldsymbol{B}^{\mathbf{W}} \mid (\boldsymbol{B}\boldsymbol{B}) \mid \boldsymbol{\mu}\left(\frac{\widehat{\delta}_{\mathbf{I}}}{\boldsymbol{T}}\right) \mid \bar{\boldsymbol{\mu}}\left(\frac{\widehat{\delta}_{\mathbf{I}}}{\widehat{\delta}_{\mathbf{I}}\boldsymbol{A}}\right) \\ V &::= x \mid y \mid z \mid \ldots \\ \mathbf{W} &::= [0,1] \subseteq \mathbb{R} \\ \mathbf{I} &::= 1 \mid 2 \end{aligned}$$

The three-polarizer paradox experiments are then expressed by context

$$\Gamma = \{x : v, \left(\frac{:\widehat{\delta}_1}{\widehat{\delta}_1 x}\right) : v \sqcup h, \left(\frac{:\widehat{\delta}_2}{\widehat{\delta}_1 x}\right) : v \sqcup h, \left(\frac{:\widehat{\delta}_2}{(:(\!|z, z|\!)/\widehat{\delta}_1 x)}\right) : v \sqcup h\}$$

where v (resp. h) stands for the photon being in the vertical (resp. horizontal) linear polarization state. One then shows that this quantum logic framework, with the corresponding τ-calculus on proof terms and system of natural deduction inference rules, yields a solution to the three-polarizer paradox, provided that some specific form of Bohr complementarity principle [2] is embedded in the structure of the new constructive quantum logic. The embedding uses a notion of *critical pair* of judgments and replaces the notion of theoremhood with membership in some *quantum extension* E of the initially given data, where $E = \bigcup_{n=0}^{\infty} E_n$ with sequence of sets $(E_n)_{n\in\mathbb{N}}$ defined by $E_0 = \Gamma_0$ and

$$\begin{aligned} E_{n+1} = Th(E_n) \quad &\cup \quad \{M : \varphi_i \mid \Gamma \vdash M : \varphi_i \text{ and } E \text{ contains no critical pair}\} \\ &\cup \quad \{N : \neg\varphi_i \mid \Gamma \vdash N : (\varphi_i \to \mathsf{F}) \text{ and } E \text{ contains no critical pair}\}. \end{aligned}$$

[1] G. Birkhoff and J. von Neumann, *The logic of quantum mechanics*, ***Annals of Mathematics***, vol. 37 (1936), pp.823–843.

[2] N. Bohr, ***Atomic theory and the description of nature***, pp.10, 52–91, Cambridge University Press, 1961.

[3] M.H.B. Sørensen and P. Urzyczyn, ***Lectures on the Curry-Howard isomorphism***, Elsevier, 2006.