▸ FAUSTINE OLIVA, *How can hypermaps guide us through the computer-assisted proof of the Four Colour Theorem?*.

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The Four Colour Theorem's proof [1] [4] is the first substantial result which has been entirely formalized and checked by the proof-assistant Coq [3]. Coq enables the user to write in one programming language — the Calculus of Inductive Constructions — both the mathematical argument and the computational part of the proof. The correctness of the proof-as-program entails the validity of the proof. Because it is hard for a computer to deal with topological arguments, the original statement of the theorem which is about maps had to be rephrased: it became a statement relying on hypermaps. Traditionally, a hypermap is a combinatorial structure defined by a couple of permutations on a finite set [2]. Our hypothesis is that understanding why the hypermap is suitable to rephrase the original problem and prove the theorem in Coq and how its properties are used in order to do so sheds a light on the design of this computer-assisted proof. First, we will show how and why we move from the topological field to the combinatorial one. Secondly, we will present how the hypermap is implemented and used as a data structure. Finally, we will focus on the structural and teleological properties of the proof that had been highlighten.

[1] Appel Kenneth, Haken Wolfgang, *Every Planar Map is Four Colorable Part I: Discharging, Part II: Reducibility*, ***Illinois Journal of Mathematics***, vol. 21 (1977), no. 3, pp. 429–567.

[2] Cori Robert, *Un code pour les graphes planaires et ses applications*, ***Astérisque***, vol. 27 (1975)

[3] Gonthier Georges, *A computer-checked proof of the Four Colour Theorem*, Year, 2005

[4] Robertson Neil, Sanders Daniel, Seymour Paul, Thomas Robin, *The Four-Color Theorem*, ***Journal of Combinatorial Theory***, vol. 70 (1997), no. 1, pp. 2–44.