▶ GITI OMIDVAR, LUTZ STRAßBURGER, *Combinatorial Flows and their compositions.*

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The modern form of proof theory was established by David Hilbert with his project on the foundations of Mathematics, famous as Hilbert's problems posed in the 19's. The 24th problem of Hilbert [8], which asks about the possibility of comparing proofs, is as old as proof theory since it is the nature of the study. So far proof theorists have two completely different approaches to this problem. The first is to find suitable proof transformations and postulate that they are the same if they can be transformed into each other using these transformations. This can be achieved via proof normalization [7] or rule permutations [6]. However, transformations such as cut normalization affect proof complexity drastically. The second approach to the problem is to define suitable canonical proof representations. The most prominent examples are λ-terms [3], proof nets [2], and combinatorial proofs [4]. Our approach, by introducing combinatorial flows, classifies as the latter among the two approaches. We introduce combinatorial flows as a graphical representation of proofs in classical propositional logic with the possibility of showing the additive and multiplicative nature of the logic. Combinatorial proofs can be seen as a generalization of atomic flows [5] or combinatorial proofs. From atomic flows they inherit the close correspondence with open deduction and deep inference and the possibility of tracing the occurrences of atoms in a derivation. From combinatorial proofs, introduced by Hughes, they inherit the correctness criterion that allows the reconstruction of the derivation from the flow. In fact, combinatorial flows form a proof system in the sense of Cook and Reckhow [1]. We show how to translate between open deduction derivations and combinatorial flows, how to trace atoms in the proof, and how combinatorial flows are related to combinatorial proofs with cuts.

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