▶ EUGENIO ORLANDELLI, AND MATTEO TESI, *A proof-theoretic approach to monadic logic.*
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Monadic logic is well-known to constitute a decidable fragment of first-order logic. The proof of the decidability result dates back to Löwenheim who employed model-theoretic methods [1]. A different proof of the same result was offered by Quine and published in the Journal of Symbolic Logic [3]. The key feature of Quine's proof is the use of a transformation of monadic formula in a certain normal form to which we refer as *innex* normal form. In essence, every monadic formula is equivalent to a formula which is a boolean combination of atomic formulas and existentially (universally) quantified finite conjunctions (disjunctions).

We propose a proof-theoretic approach to the issue. In particular, we first give a detailed reconstruction of Quine's procedure to reduce formulas in innex normal form via cuts and provable equivalences. Next, we introduce a sequent calculus for formulas in innex normal form which enjoys strong termination of the proof search in the sense that every bottom-up sequence of applications of the rules terminates leading to a derivation or to a finite countermodel.

A thorough structural analysis of the system is carried out by showing the admissibility of the structural rules of weakening, contraction and cut. The analysis is interesting as it is peculiar of the system due to certain context-restrictions imposed on the rules. Also, the cut-elimination strategy is obtained arguing by a single inductive parameter - the degree of the cut formula - rather than by induction on lexicographically ordered pairs - the degree of the cut formula and the sum of the height of the derivations of the premises of the cut - as usual in first-order languages (a proof running by induction on a single inductive parameter - different from the degree of the cut formula - for classical propositional logic can be found in [2]).

Finally, applications to the theory of syllogisms are described and themes for future research are briefly sketched.

[1] L. Löwenheim, *On possibilities in the calculus of relatives*, ***A Source Book in Mathematical Logic*** (Jean van Heijenoort, editor), Harvard University Press, Publisher's address, 1967.

[2] G. Pulcini, *A note on cut-elimination for classical propositional logic*, Archive for Mathematical Logic, vol. 61, pp. 555–565, 2022.

[3] W.V.O. Quine , *On the logic of quantification*, ***The Journal of Symbolic Logic***, vol. 10 (1945), no. 1, pp. 1–12.