

- LUIZ CARLOS PEREIRA, ELAINE PIMENTEL, AND VALERIA DE PAIVA, *Translations and Prawitz ecumenical system*.

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Ecumenical systems are formal codifications where two or more logics can co-exist in peace, which means that these logics accept and reject the same formulae, the same rules and the same basic principles. Dag Prawitz proposed a natural deduction ecumenical system [2], where classical logic and intuitionistic logic are codified in the same system (see also [1]). In this system, the classical logician and the intuitionistic logician would share the universal quantifier, conjunction, negation and the constant for the absurd, but they would each have their own existential quantifier, disjunction and implication, with different meanings. Prawitz main idea is that these different meanings are given by a semantic framework that can be accepted by both parties. The rules for the intuitionistic operators ($\rightarrow_i, \vee_i, \exists_i$) and for the shared operators ($\wedge, \neg, \perp, \forall$) are the usual Gentzen-Prawitz natural deduction introduction and elimination rules. The rules for the classical propositional operators are as follows:

$$\begin{array}{c}
 \frac{[A] \quad [\neg B] \quad \Pi}{\perp} \\
 \frac{\perp}{(A \rightarrow_c B)} \rightarrow_c\text{-int}
 \end{array}
 \qquad
 \frac{(A \rightarrow_c B) \quad A \quad \neg B}{\perp}$$

$$\frac{[\neg A] \quad [\neg B] \quad \Pi}{\perp} \vee_c\text{-int}
 \qquad
 \frac{(A \vee_c B) \quad \neg A \quad \neg B}{\perp}$$

This short note has two main objectives. The first is to show, in the propositional case, that there are interesting relations between the Gödel-Gentzen translation and the ecumenical perspective, but that the later cannot be reduced to the former. The second main objective is to investigate the possibility of ecumenical systems with two independent negations, one classical and one intuitionistic.

[1] PIMENTEL, E., PEREIRA, LUIZ C. AND DE PAIVA, VALERIA , *An ecumenical notion of entailment (2020)*, *Synthese*, vol. 198, (2019), pp.5391-5413.

[2] PRAWITZ, D., *Classical versus intuitionistic logic*, *Why is this a Proof?*, *Festschrift for Luiz Carlos Pereira* (Hermann Haesler, Wagner Sanz, and Bruno Lopes editors), College Books, UK, 2015, pp. 15–32.