▸ PHILIPP PROVENZANO, *Extracting ω-models from well-ordering principles.*
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It is known by works of Girard that arithmetical comprehension is equivalent to the following well-ordering principle: $\forall\alpha.\,\mathrm{WO}(\alpha) \to \mathrm{WO}(\omega^{\alpha})$, where $\omega^{\bullet}$ denotes an exponentiation operation on linear orders. Iterating this principle establishes a connection between preservation of well-orders by the Veblen function φ, which can be viewed as transfinitely iterated exponentiation, to corresponding comprehension schemes $\Pi^0_\alpha - \mathrm{CA}_0$, denoting α-iterated Π^0_1 comprehension. The latter theory is more commonly known as ATR_0 and it is a famous result by Friedman that it is equivalent to the well-ordering principle $\forall\alpha.\,\mathrm{WO}(\alpha) \to \mathrm{WO}(\varphi_\alpha(0))$. Relativizations of this iteration principle, starting from arbitrary sufficiently well-behaved transformations of well-orders, have been studied in my master thesis, using ordinal analysis as a proof-theoretic tool. The goal of this talk is to give a more direct computability theoretic account of the result.
The comprehension principles $\Pi^0_\alpha - \mathrm{CA}_0$ can be more conveniently stated as iterated existence of ω-models. These are models of arithmetic with standard first-order domain and countable set-universe. The principle $\omega\,\mathrm{Con}(T)$ states that ω-models of a theory T exist relative to any given set. Iterating this leads to the principles

$$\omega\,\mathrm{Con}^{\alpha}(T) := \omega\,\mathrm{Con}\left(T + \left\{\omega\,\mathrm{Con}^{\beta}(T) \,\middle|\, \beta \prec \alpha\right\}\right)$$

and we get $\Pi^0_{\omega^{1+\alpha}} - \mathrm{CA}_0 \equiv \omega\,\mathrm{Con}^{\alpha}(\mathrm{ACA}_0)$. For a sufficiently well-behaved transformation D of well-orders, our main result can then be stated as the equivalence between the principle of $D^{\bullet}$ preserving well-orders and

$$\forall\alpha.\,\mathrm{WO}(\alpha) \to \omega\,\mathrm{Con}^{\alpha}(\mathrm{ACA}_0 + \text{"}D \text{ preserves well-orders"}),$$

where $D^{\bullet}$ denotes a suitably defined transfinite iteration of D. In order to give a direct proof of this result, we want to reduce the task of finding a universe for such ω-models to the computational problem of finding an infinite path in α when given an infinite path in $D^{\alpha}(0)$ for a linear order α. The latter is just the computational content of the statement that $D^{\bullet}(0)$ preserves well-orders.
For the case of $D = \omega^{\bullet}$, this has been answered by Marcone and Montalbán in [1]. I show how their method relativizes to arbitrary transformations of well-orders by associating a corresponding set-existence operator.

[1] Alberto Marcone, Antonio Montalbán, *The Veblen functions for computability theorists*, ***The Journal of Symbolic Logic***, vol. 76 (2011), no. 2, pp. 575–602.