

- SAM SANDERS, *The Biggest Five of Reverse Mathematics*.

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I provide an overview of joint work with Dag Normann on the higher-order Reverse Mathematics (RM for short) of the Big Five systems and the surprising limits of this enterprise ([3]).

The well-known *Big Five phenomenon* of RM is the observation that a large number of theorems from ordinary mathematics are either provable in the base theory or equivalent to one of only four systems; these five systems together are called the ‘Big Five’ of RM. The aim of this paper is to **greatly** extend the Big Five phenomenon, working in Kohlenbach’s *higher-order* RM ([1]).

In particular, we have established numerous equivalences involving the **second-order** Big Five systems on one hand, and well-known **third-order** theorems from analysis about (possibly) discontinuous functions on the other hand. We both study relatively tame notions, like *cadlag* or *Baire 1*, and potentially wild ones, like *quasi-continuity*. We also show that *slight* generalisations and variations (involving e.g. the notions *Baire 2* and *cliquishness*) of the aforementioned third-order theorems fall *far* outside of the Big Five. In particular, these slight generalisations and variations imply the principle  $\text{NIN}_{[0,1]}$  from [2], i.e. there is no injection from  $[0, 1]$  to  $\mathbb{N}$ . We have no explanation for this phenomenon.

#### REFERENCES.

- [1] Ulrich Kohlenbach, *Higher order reverse mathematics*, Reverse mathematics 2001, Lect. Notes Log., vol. 21, ASL, 2005, pp. 281–295.
- [2] Dag Normann and Sam Sanders, *On the uncountability of  $\mathbb{R}$* , Journal of Symbolic Logic, DOI: [doi.org/10.1017/jsl.2022.27](https://doi.org/10.1017/jsl.2022.27) (2022), pp. 43.
- [3] ———, *The Biggest Five of Reverse Mathematics*, Submitted, arxiv: <https://arxiv.org/abs/2212.00489> (2023), pp. 39.