▸ ANDREI SIPOŞ, *The computational content of super strongly nonexpansive mappings.*

Research Center for Logic, Optimization and Security (LOS), Department of Computer Science, Faculty of Mathematics and Computer Science, University of Bucharest, Academiei 14, 010014 Bucharest, Romania.
Simion Stoilow Institute of Mathematics of the Romanian Academy, Calea Griviţei 21, 010702 Bucharest, Romania.
Institute for Logic and Data Science, Popa Tatu 18, 010805 Bucharest, Romania.
E-mail: `andrei.sipos@fmi.unibuc.ro`.
URL Address: `https://cs.unibuc.ro/~asipos/`.

Strongly nonexpansive mappings are a core concept in convex optimization. Recently, they have begun to be studied from a quantitative viewpoint: U. Kohlenbach has identified in [2] the notion of a 'modulus' of strong nonexpansiveness, which leads to computational interpretations of the main results involving this class of mappings (e.g. rates of convergence, rates of metastability). This forms part of the greater research program of 'proof mining', initiated by G. Kreisel and highly developed by U. Kohlenbach and his collaborators, which aims to apply proof-theoretic tools to extract computational content (which may not be immediately apparent) from ordinary proofs in mainstream mathematics (for more information on the current state of proof mining, see the book [1] and the recent survey [3]). The quantitative study of strongly nonexpansive mappings has later led to finding rates of asymptotic regularity for the problem of 'inconsistent feasibility' [4, 7], where one essential ingredient has been a computational counterpart of the concept of rectangularity, recently identified in [5] as a 'modulus of uniform rectangularity'.

Last year, Liu, Moursi and Vanderwerff [6] have introduced the class of 'super strongly nonexpansive mappings', and have shown that this class is tightly linked to that of uniformly monotone operators. What we do is to provide a modulus of super strong nonexpansiveness, give examples of it in the cases e.g. averaged mappings and contractions for large distances and connect it to the modulus of uniform monotonicity. In the case where the modulus is supercoercive, we give a refined analysis, identifying a second modulus for supercoercivity, specifying the necessary computational connections and generalizing quantitative inconsistent feasibility.

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[3] ——— *Proof-theoretic methods in nonlinear analysis*, ***Proceedings of the International Congress of Mathematicians 2018*** (B. Sirakov, P. Ney de Souza and M. Viana, editors), vol. 2, World Scientific, 2019, pp. 61–82.

[4] ——— *A polynomial rate of asymptotic regularity for compositions of projections in Hilbert space.* ***Foundations of Computational Mathematics*** 19, no. 1, 83–99, 2019.

[5] U. Kohlenbach, N. Pischke, *Proof theory and nonsmooth analysis.* Preprint, 2022. To appear in: ***Philosophical Transactions of the Royal Society A***.

[6] L. Liu, W. M. Moursi, J. Vanderwerff, *Strongly nonexpansive mappings revisited: uniform monotonicity and operator splitting.* arXiv:2205.09040 [math.OC], 2022.

[7] A. Sipoş, *Quantitative inconsistent feasibility for averaged mappings.* ***Optimization Letters*** 16, no. 6, 1915–1925, 2022.