

- JUAN M SANTIAGO SUÁREZ, MATTEO VIALE, *Boolean valued semantics for infinitary logics.*

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It is well known that the completeness theorem for $L_{\omega_1\omega}$ fails with respect to Tarski semantics. Mansfield [1] showed that it holds for $L_{\infty\infty}$ if one replaces Tarski semantics with boolean valued semantics. We use forcing to improve his result in order to obtain a stronger form of boolean completeness (but only for $L_{\infty\omega}$). Leveraging on our completeness result, we establish the Craig interpolation property and a strong version of the omitting types theorem for $L_{\infty\omega}$ with respect to boolean valued semantics. We also show that a weak version of these results holds for $L_{\infty\infty}$ (if one leverages instead on Mansfield's completeness theorem). Furthermore we bring to light (or in some cases just revive) several connections between the infinitary logic $L_{\infty\omega}$ and the forcing method in set theory.

[1] RICHARD MANSFIELD, *The Completeness Theorem for Infinitary Logic*, *The Journal of Symbolic Logic*, Vol. 37, No. 1 (Mar., 1972), pp. 31-34.