

- HSING-CHIEN TSAI, ZE-YUAN DUAN, *On the Complexity of First-order Axiomatizable Mereological Theories.*

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In this talk, we are only concerned with first-order mereological theories which can be axiomatized by using the following list of axioms that can be found in the literature.

(P1: reflexivity)  $\forall x Pxx$

(P2: anti-symmetry)  $\forall x \forall y ((Pxy \wedge Pyx) \rightarrow x = y)$

(P3: transitivity)  $\forall x \forall y \forall z ((Pxy \wedge Pyz) \rightarrow Pxz)$

(EP: extensionality)  $\forall x \forall y (\exists z PPzx \rightarrow (\forall z (PPzx \leftrightarrow PPzy) \rightarrow x = y))$

(WSP: weak supplementation)  $\forall x \forall y (PPxy \rightarrow \exists z (PPzy \wedge \neg Ozx))$

(SSP: strong supplementation)  $\forall x \forall y (\neg Pyx \rightarrow \exists z (Pzy \wedge \neg Ozx))$

(FS: finite sum)  $\forall x \forall y (Uxy \rightarrow (\exists z \forall w (Owz \leftrightarrow (Owx \vee Owy))))$

(FP: finite product)  $\forall x \forall y (Oxy \rightarrow \exists z \forall w (Pwz \leftrightarrow (Pwx \wedge Pwy)))$

(A: atomicity)  $\forall x \exists y (Pyx \wedge \neg \exists z PPzy)$ , where  $y$  is an “atom”, for it has no proper part.

(AL: atomlessness)  $\forall x \exists y PPyx$

(G: existence of the greatest member)  $\exists x \forall y Pyx$

(C: complementation)  $\forall x (\neg \forall z Pzx \rightarrow \exists z \forall w (Pwz \leftrightarrow \neg Owz))$

(UF: unrestricted fusion axiom schema)  $\exists x \alpha \rightarrow \exists z \forall y (Oyz \leftrightarrow \exists x (\alpha \wedge Oyx))$ , for any formula  $\alpha$  where  $z$  and  $y$  do not occur free.

Previously, we have shown the following facts, where **CEM** is the theory axiomatized by (P1), (P2), (P3), (SSP), (FS) and (FP).

- (1) Any theory strictly weaker than **CEM**+(C) is finitely inseparable and hence undecidable. (2) Any theory weaker than **CEM**+(G) is undecidable, but **CEM**+(G) is not finitely inseparable. (3) Any theory stronger than **CEM**+(C) is decidable.

In this talk, we will see that the undecidable theories mentioned in (1) or (2) are 1-complete and the decidable theories mentioned in (3) are NP-hard. Moreover, we did not deal with the atomless theories previously. Now we will show that any theory weaker than **CEM**+(G)+(AL) is 1-complete (note that an atomless theory cannot be finitely inseparable).