

- CHENG-SYUAN WAN, *Towards Skew Non-Commutative MILL with Additives*.

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This work concerns the proof theory of (left) skew monoidal categories and their variants (e.g. closed monoidal, symmetric monoidal), continuing the line of work initiated in recent years by Uustalu et al. [2, 3, 4, 5]. Skew monoidal categories are a weak version of Mac Lane’s monoidal categories, where the structural laws  $\lambda$ ,  $\rho$ , and  $\alpha$  are not required to be invertible, they are merely natural transformations with a specific orientation. Uustalu et al. describe sequent calculi for numerous variants of skew monoidal categories, which can be identified as restricted substructural fragments of intuitionistic linear logic. These calculi enjoy cut elimination and admit a focusing strategy, sharing resemblance with Andreoli’s normalization technique for linear logic [1]. The focusing procedure is useful for solving the coherence problem of the considered categories with skew structure.

Here we investigate possible extensions of the sequent calculi of Uustalu et al. with additive connectives. As a first step, we extend the sequent calculus in [4] with additive conjunction and disjunction ( $\wedge$  and  $\vee$ ), corresponding to studying the proof theory of skew monoidal categories with binary Cartesian products and coproducts. We introduce a new focused sequent calculus of derivations in normal form, which employs tag annotations in the style of [2] to reduce non-deterministic choices in bottom-up proof search. Apart from statements and proofs on pen and paper, we also want to formalize the focused sequent calculus and verify its correctness in the Agda proof assistant. We believe this to be beneficial for the development of modular normalization techniques for substructural logics arising as an extension of our sequent calculus, e.g. full Lambek calculus or intuitionistic linear logic.

This is work in progress in collaboration with Niccolò Veltri (Tallinn University of Technology).

[1] JEAN-MARC ANDREOLI, *Logic Programming with Focusing Proofs in Linear Logic*, *Journal of Logic and Computation*, vol.2(3) (1992), pp. 297–347.

[2] TARMO UUSTALU, NICCOLÒ VELTRI, AND CHENG-SYUAN WAN, *Proof Theory of Skew Non-Commutative MILL*, *Electronic Proceedings in Theoretical Computer Science*, vol.358 (2022), pp. 118–135.

[3] TARMO UUSTALU, NICCOLÒ VELTRI, AND NOAM ZEILBERGER, *Deductive Systems and Coherence for Skew Prounital Closed Categories*, *Electronic Proceedings in Theoretical Computer Science*, vol.332 (2021), pp. 35–53.

[4] TARMO UUSTALU, NICCOLÒ VELTRI, AND NOAM ZEILBERGER, *The Sequent Calculus of Skew Monoidal Categories*, *Joachim Lambek: The Interplay of Mathematics, Logic, and Linguistics* (2021), pp. 377–406.

[5] NICCOLÒ VELTRI, *Coherence via Focusing for Symmetric Skew Monoidal Categories*, *Lecture Notes in Computer Science*, vol.13028 (2021), pp. 184–200.