

- ANDREAS WEIERMANN, *The phase transition for Harvey Friedman's Bolzano Weierstrass principle.*

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Let f be a weakly monotone and unbounded number-theoretic function. Harvey Friedman's Bolzano Weierstrass principle with respect to f is the following assertion (BW_f). $(\forall K \geq 3)(\exists M)(\forall x_1, \dots, x_M \in [0, 1])(\exists k_1, \dots, k_K (k_1 < \dots < k_K \leq M \wedge (\forall l \leq K - 2) |x_{k_{l+1}} - x_{k_{l+2}}| < \frac{1}{f(k_l)}))$. Friedman has shown that BW_f is true (by an application of the compactness of the Hilbert cube). Moreover Friedman has shown that for $f(x) = 1/x^{1+\varepsilon}$ where $\varepsilon > 0$ the principle BW_f is not provable from $I\Sigma_1$. He also has shown that for $f(x) = \log(x)/x$ the assertion BW_f is provable from $I\Sigma_1$ and asked for the strength of BW_f for $f(x) = 1/x$ and $f(x) = 1/(x \log(x))$.

In our talk we answer these two questions and we give rather sharp bounds on the phase transition window for those functions f for which BW_f is provable or unprovable from $I\Sigma_1$. We also discuss the Friedman principle for monotone increasing sequences.

Finally as a real analysis spin off we obtain explicit formulas for the derivative of the smooth version of the inverse function of the inverses of the d -th branch of the Ackermann function for any natural number d .