▸ BARTOSZ WIĘCKOWSKI, *Towards a modal proof theory for reasoning from counterfactual assumptions.*

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In current research on structural proof theory, counterfactual inference is typically studied from a model-theoretic perspective. On this perspective, possible worlds models are methodologically basic. Model-theoretically defined consequence relations come first, and structural proof systems, usually transmitted via Hilbert-style axiom systems, have to be defined for these consequence relations. Structural proof theory is thus methodologically secondary. Labelled (or external) proof systems for counterfactual logics which incorporate possible worlds structures into their rules (e.g., [2, 3]) clearly illustrate this model-theoretic dependency. Importantly, the logics usually extend classical logic. By contrast, on the proof-theoretic perspective on counterfactual inference, we start from a certain primacy of inferential practice and proof theory. Proof-theoretic structure comes first. Meaning is explained in terms of proofs ([5]). Models are required neither for the formal explanation of the meaning of counterfactuals nor for that of counterfactual inference. Taking a proof-theoretic perspective and a constructive stance on meaning and truth (cf. BHK), we extend the rudimentary intuitionistic subatomic natural deduction system for counterfactual implication presented in [8] with rules for conjunction. This proof system is *modal* insofar as derivations in it make use of *modes* of assumptions which are sensitive to the factuality status (factual, counterfactual, independent) of the formula that is to be assumed. This status is determined by means of a so-called reference proof system on top of which the modal proof system is defined. Specifically, the factuality status of atomic sentences is determined by the subatomic system (cf. [6]) of the reference system. The introduction and elimination rules for counterfactual implication and conjunction draw on this status. We establish normalization (cf. [4]) and the subexpression (hence, subformula) property for the system. On the basis of these results, we define a proof-theoretic semantics for counterfactual implication and conjunction, discuss the internal completeness (cf. [1]) of the system, and use the method of counter-derivations ([7]) to assess some familiar counterfactual fallacies and logics.

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