

- ALEKSI ANTTILA, MATILDA HÄGGBLUM, AND FAN YANG, *Axiomatizing modal inclusion logic*.

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We present a natural deduction axiomatization of modal inclusion logic. Modal inclusion logic is the modal variant of first-order inclusion logic, introduced by Galliani [1]. Like dependence logic [3] (Väänänen 2007), inclusion logic adopts team semantics, in which formulas are evaluated with respect to sets of evaluation points (teams) rather than single points. For modal logics with team semantics, teams are sets of possible worlds in a Kripke model. Modal inclusion logic is classical modal logic extended with inclusion atoms, which describe that the possible truth values a sequence of formulas can obtain in a team are also truth values another sequence of formulas has somewhere in the team.

We review the proof of Hella and Stumpf [2] that modal inclusion logic is expressively complete for classes of Kripke models with teams that are closed under unions, closed under k -bisimulation for some natural number k , and have the empty team property. Through the expressive completeness proof, we obtain a normal form for formulas in modal inclusion logic, which plays a central role in the proof of the completeness theorem. Our proof system for modal inclusion logic builds on the proof system defined for propositional inclusion logic by Yang [4].

[1] PIETRO GALLIANI, *Inclusion and exclusion dependencies in team semantics - on some logics of imperfect information*, ***Annals of Pure and Applied Logic***, vol. 163 (2012), no. 1, pp. 68–84.

[2] LAURI HELLA, JOHANNA STUMPF, *The expressive power of modal logic with inclusion atoms*, ***International Symposium on Games, Automata, Logics and Formal Verification*** vol. 6 (2015), pp. 129–143.

[3] JOUKO VÄÄNÄNEN, *Dependence logic: a new approach to independence friendly logic*, London Mathematical Society student texts, Cambridge University Press, 2007.

[4] FAN YANG, *Propositional union closed team logics*, ***Annals of Pure and Applied Logic***, vol. 173 (2022), no. 6, 103102.