▸ S. BONZIO AND M. PRA BALDI, *On the structure of Bochvar algebras.*
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The recent years have seen a renaissence of interests and studies around weak Kleene logics, logical formalisms that were considered, in the past, perhaps less attractive in the panorama of three-valued logics, due to infectious behavior of the third-value. This late (re)discovery regards, almost exclusively, *internal* rather than *external* logics. The latter, in essence consisting of linguistic expansions of the former, includes Bochvar external logic (introduced in [1]) and the external version of Paraconsistent weak Kleene (introduced by Segerberg [11]). Bochvar external logic B_e is defined in the algebraic language $\mathcal{L}$: $\langle \neg, \vee, \wedge, J_0, J_1, J_2, 0, 1\rangle$ (of type $(1, 2, 2, 1, 1, 1, 0, 0)$), as the logic induced by the single matrix $\langle \mathbf{WK}^e, \{1\}\rangle$ (whose algebraic reduct is recalled in Fig. 1).

	¬
1	0
1/2	1/2
0	1

∨	0	12	1
0	0	1/2	1
1/2	1/2	1/2	1/2
1	1	1/2	1

∧	0	1/2	1
0	0	1/2	0
1/2	1/2	1/2	1/2
1	0	1/2	1

φ	$J_0\varphi$
1	0
1/2	0
0	1

φ	$J_1\varphi$
1	0
1/2	1
0	0

φ	$J_2\varphi$
1	1
1/2	0
0	0

FIGURE 1. The algebra $\mathbf{WK}^e$.

Finn and Grigolia [4, 5] introduced a Hilbert-style axiomatization for B_e and also the class of Bochvar algebras.

DEFINITION 1. A Bochvar algebra $\mathbf{A} = \langle A, \vee, \wedge, \neg, J_0, J_1, J_2, 0, 1\rangle$ is an algebra of type $\langle 2, 2, 1, 1, 1, 1, 0, 0\rangle$ satisfying the following identities and quasi-identities:

1. $x \vee x \approx x$;
2. $x \vee y \approx y \vee x$;
3. $(x \vee y) \vee z \approx x \vee (y \vee z)$;
4. $x \wedge (y \vee z) \approx (x \wedge y) \vee (x \wedge z)$;
5. $\neg(\neg x) \approx x$;
6. $\neg 1 \approx 0$;
7. $\neg(x \vee y) \approx \neg x \wedge \neg y$;
8. $0 \vee x \approx x$;
9. $J_2 J_k x \approx J_k x$, for every $k \in \{0, 1, 2\}$;
10. $J_0 J_k x \approx \neg J_k x$, for every $k \in \{0, 1, 2\}$;
11. $J_1 J_k x \approx 0$, for every $k \in \{0, 1, 2\}$;
12. $J_k(\neg x) \approx J_{2-k} x$, for every $k \in \{0, 1, 2\}$;
13. $J_i x \approx \neg(J_j x \vee J_k x)$, for $i \neq j \neq k \neq i$;
14. $J_k x \vee \neg J_k x \approx 1$, for every $k \in \{0, 1, 2\}$;

15. $((J_i x \vee J_k x) \wedge J_i x) \approx J_i x$, for $i, k \in \{0, 1, 2\}$;
16. $x \vee J_k x \approx x$, for $k \in \{1, 2\}$;
17. $J_0(x \vee y) \approx J_0 x \wedge J_0 y$;
18. $J_2(x \vee y) \approx (J_2 x \wedge J_2 y) \vee (J_2 x \wedge J_2 \neg y) \vee (J_2 \neg x \wedge J_2 y)$;
19. $J_0 x \approx J_0 y \ \&\ J_1 x \approx J_1 y \ \&\ J_2 x \approx J_2 y \ \Rightarrow\ x \approx y$.

The class $\mathcal{BCA}$ of Bochvar algebras forms a proper quasi-variety. Only recently [2], it has been shown that $\mathcal{BCA}$ algebrizes Bochvar external logic.

In this contribution, we show that the algebraic construction of the Płonka sum [7, 8] (more comprehensive expositions are [9], [3, Ch. 2]) allows to provide a representation theorem for Bochvar algebras. Such a representation allows us to characterize logical filters and to provide a constructive proof of the that the $\mathcal{BCA}$ is generated by the single algebra $\mathbf{WK}^e$.[1]Moreover, we describe the lattice of (non-trivial) subquasivarieties of Bochvar algebras and, dually, the lattice of extensions of B_e, which consist of a three-elements chain, and prove that B_e is not passively structurally complete, while its non-trivial extension – here named NB_e – is structurally complete (see [6] and [10]). In the final part of the talk, we will focus on relevant algebraic properties for algebraizable logics and, upon relying once more on the Płonka sum representation theorem, we show that $\mathcal{BCA}$ has surjective epimorphisms and the amalgamation property.

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[1] This can be derived also from general results on algebraizable logics.