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The modal μ -calculus \mathcal{L}_μ is a decidable, but very expressive extension of modal logic as it embeds many modal/temporal logics such as PDL, CTL and CTL*. Dawar and Otto [1] have shown a modal characterisation theorem for \mathcal{L}_μ over **K4** frames. In particular, they proved that a weaker logic $\mathcal{L}_{\diamond^\infty}$ is equivalent to the bisimulation invariant part of *MSO*. The tangle logic $\mathcal{L}_{\diamond^\infty}$ is defined by replacing the fixed point operator with the tangle derivative operator which is defined on a finite set of formulas Γ as $\diamond^\infty \Gamma := \nu x. \bigwedge_{\varphi \in \Gamma} \diamond(\varphi \wedge x)$. It is of significance in the study of topological modal logic as it corresponds to an extension of the notion of the perfect core of a set. Furthermore, similar questions of the expressivity of tangle operators arise for arbitrary topological derivative spaces which are the topological semantics characterised by the logic of so-called weakly transitive frames **wK4**.

An analysis of Baltag, Bezhanishvili and Fernández-Duque [2] gave completeness and weak FMP for \mathcal{L}_μ over **wK4** frames as well as topological derivative spaces. This was done by using the notion of finality, where a world w is final in a model M for some formula ϕ if $M, w \models \phi$ and given u with wRu and $M, u \models \phi$ then uRw . They show that we cannot have the same kind of modal characterisation theorem of \mathcal{L}_μ over **wK4**.

At the same time we know that $\mathcal{L}_{\diamond^\infty}$ is not as expressive as \mathcal{L}_μ over **wK4** frames. We use the notion of finality and the finite model property to show that a slightly more expressive tangle operator does in fact provide a logic equivalent to that of \mathcal{L}_μ over **wK4** frames. As a corollary we get a model theoretic proof of the collapse of the alternation hierarchy over **wK4** frames, a result which was recently shown using proof theoretic methods in [3].

[1] ANUJ DAWAR AND MARTIN OTTO, *Modal characterisation theorems over special classes of frames*, *Annals of Pure and Applied Logic*, vol. 161 (2009), no. 1, pp. 1-42.

[2] BALTAG, ALEXANDRU AND BEZHANISHVILI, NICK AND FERNÁNDEZ-DUQUE, DAVID, *The Topological Mu-Calculus: completeness and decidability*, *2021 36th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS)*

[3] PACHECO, LEONARDO AND TANAKA, KAZUYUKI, *The Alternation Hierarchy of the μ -calculus over Weakly Transitive Frames*, *Logic, Language, Information, and Computation*, (2022), pp. 207-220.