LOGIC COLLOQUIUM 2023

European Summer Meeting
of the
Association for Symbolic Logic

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Book of Abstracts

Table of Contents

Abstracts of the Talks by the Invited Speakers

Logic Colloquium 2023

Contents

Gal Binyamini – Unlikely intersections: a mathematical theory of strange coincidences. An "unlikely intersection" problem is one where the number of constraints strictly exceeds the degrees of freedom. For such problems, the existence of a solution can be thought of as an unlikely "coincidence". A general paradigm in this area is that if a system exhibits many coincidences, then there must be some hidden structure in the system that forces them to occur – things happen for a reason.

Many classical problems in arithmetic geometry can be viewed as unlikely intersection problems. I'll discuss some of these examples along with other unlikely intersection problems that come up in analysis and dynamics. Surprisingly, many of these problems - even those having seemingly nothing to do with algebraic data - often turn out to be fundamentally linked to the study of integer and rational points in certain sets. I'll explain how logic and model theory facilitate this unexpected translation and provide very powerful tools in pursuit of the general paradigm above.

▸ GAL BINYAMINI, *Unlikely intersections: a mathematical theory of strange coincidences.*

Nicola Gambino – Two-dimensional categorical logic. Categorical logic, founded by Lawvere in the '60s, is generally concerned with the interplay between logic and category theory, with applications in both directions. In recent years, motivation from various angles, including theoretical computer science, has led to first steps in the creation of two-dimensional categorical logic, in which ordinary set-based structures are replaced by category-based ones (*e.g.* equivalence relations are replaced by groupoids), very much in analogy with the research program of categorification in algebra.

After reviewing the basics of categorical logic and outlining the key aspects of two-dimensional categorical logic, I will focus on an illustrative example, namely the 2-category of analytic functors, introduced in [3] and studied further in [4]. This 2-category possesses a wealth of structure, thus giving a good indication of the potential and complexity of two-dimensional categorical logic. In particular, it provides a model of the differential λ-calculus [1], an extension of the λ-calculus with a differential operator, in which it is possible to approximate λ-terms by a form of the Taylor series expansion [2].

[1] T. Ehrhard and L. Regnier, The differential lambda-calculus, *Theoretical Computer Science*, 309, 2003, pp. 1–41.

[2] T. Ehrhard and L. Regnier, Uniformity and the Taylor expansion of ordinary lambda terms. *Theoretical Computer Science*, 403 (2–3), 2008, pp. 347–372

[3] M. Fiore, N. Gambino, M. Hyland and G. Winskel, The cartesian closed bicategory of generalised species of structures, *Journal of the London Mathematical Society*, 77 (2) 2008, pp. 203–220.

[4] N. Gambino and A. Joyal, On operads, bimodules and analytic functors, *Memoirs of the American Mathematical Society*, 249 (1184), 2017, pp. 1–110.

▸ NICOLA GAMBINO, *Two-dimensional categorical logic*.
Department of Mathematics, University of Manchester.
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Gabriel Goldberg – Large cardinals and the Ultrapower Axiom. Gödel's constructible universe L provides a canonical model of ZFC in which one can study classical set theory without encountering unsolvable problems: typically, one cannot expect a given set theoretic question to be answerable assuming just the ZFC axioms, but one can always answer the relativization of the question to L. In one respect, however, L is defective: some of the most commonly used set theoretic hypotheses, large cardinal axioms, do not hold in L, so the constructible universe cannot be used to understand them. One of major projects in modern set theory is the inner model program, which seeks to construct canonical models generalizing of the constructible universe and satisfying large cardinal axioms. Such generalizations have been obtained for large cardinal axioms well into the hierarchy of Woodin cardinals, but it remains open whether it is possible to extend the pattern further – for example, to supercompact cardinals. The subject of this talk is the Ultrapower Axiom (UA), a set theoretic principle that abstracts some of the large cardinal combinatorics of canonical models of set theory. UA holds in any model built by anything like the current methodology of inner model theory, so by developing the theory of supercompact cardinals under the assumption of UA, one can obtain information about how to build canonical models at this level, or perhaps rule out that such a model exists at all. I'll discuss the theory of supercompact cardinals under UA developed in my PhD thesis and more recent results applying these ideas to other problems in set theory.

▸ GABRIEL GOLDBERG, *Large cardinals and the Ultrapower Axiom.*
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Martino Lupini – Definable refinements of classical algebraic invariants. In this talk I will explain how methods from logic allow one to construct refinements of classical algebraic invariants that are endowed with additional topological and descriptive set-theoretic information. This approach brings to fruition initial insights due to Eilenberg, Mac Lane, and Moore (among others) with the additional ingredient of recent advanced tools from logic. I will then present applications of this viewpoint to invariants from a number of areas in mathematics, including operator algebras, group theory, algebraic topology, and homological algebra.

▸ MARTINO LUPINI, *Definable refinements of classical algebraic invariants.*
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Igor C. Oliveira – Meta-Mathematics of Computational Complexity Theory. Despite significant efforts from computer scientists and mathematicians, the P vs. NP problem and other fundamental questions about the complexity of computations remain out of reach for existing techniques. The difficulty of making progress on such problems has motivated a number of researchers to investigate the logical foundations of computational complexity.

Over the last few decades, several works at the intersection of logic and complexity theory showed that certain fragments of Peano Arithmetic collectively known as Bounded Arithmetic can formalize a large fraction of results from algorithms and complexity (e.g., the PCP Theorem [6] and complexity lower bounds against restricted classes of Boolean circuits [5]). It is natural to consider if the same theories can settle longstanding problems about the inherent difficulty of computations (see [7, 8, 3] for some early papers on this topic).

In this talk, I will discuss the unprovability of certain statements of interest to complexity theory in theories of Bounded Arithmetic [1, 2, 4] and mention a few related open problems.

[1] Bydzovsky, Jan and Krajícek, Jan and Oliveira, Igor C., *Consistency of circuit lower bounds with bounded theories*, ***Logical Methods in Computer Science***, vol. 16 (2020), no. 2.

[2] Carmosino, Marco and Kabanets, Valentine and Kolokolova, Antonina and Oliveira, Igor C., *LEARN-uniform circuit lower bounds and provability in bounded arithmetic*, ***Symposium on Foundations of Computer Science* (FOCS'2021)**.

[3] Cook, Stephen A. and Krajíček, Jan, *Consequences of the provability of* $\mathsf{NP} \subseteq \mathsf{P/poly}$, ***Journal of Symbolic Logic***, vol. 72 (2007), no. 4, pp. 1353–1371.

[4] Li, Jiatu and Oliveira, Igor C., *Unprovability of strong complexity lower bounds in bounded arithmetic*, ***Symposium on Theory of Computing* (STOC'2023)**.

[5] Müller, Moritz and Pich, Ján, *Feasibly constructive proofs of succinct weak circuit lower bounds*, ***Annals of Pure and Applied Logic***, vol. 171 (2020), no. 2.

[6] Pich, Ján, *Logical strength of complexity theory and a formalization of the PCP theorem in bounded arithmetic*, ***Logical Methods in Computer Science***, vol. 11 (2015), no. 2.

[7] Razborov, Alexander A., *Bounded arithmetic and lower bounds in Boolean complexity*, ***Feasible Mathematics II*** (Clote, Peter and Remmel, Jeffrey, editors), Birkhäuser, 1995, pp. 344–386.

[8] Razborov, Alexander A., *Unprovability of lower bounds on circuit size in certain fragments of bounded arithmetic*, ***Izvestiya: Mathematics***, vol. 59 (1995), no. 1, pp. 205.

▸ IGOR C. OLIVEIRA, *Meta-Mathematics of Computational Complexity Theory.*
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Francesca Poggiolesi – Explanatory derivations in first-order logic. To explain phenomena in the world, to answer the question "why" rather than the question "what", is one of the central human activities and one of the main goals of rational inquiry. There are several types of explanation: one can explain by drawing an analogy, as one can explain by dwelling on the cause of a certain phenomenon (see e.g. see [4]). Amongst these different kinds of explanation, in the last decade philosophers have become receptive to those explanations which explain by providing *the reasons why* a statement is true; these explanations are called "non-causal" or "conceptual" explanations (e.g. see [1]). Conceptual explanations derive their explanatory power from a network of conceptual relations and for this feature, they are *prime facie* a natural object for logical analysis. The main aim of the talk is to propose a logical account of conceptual explanations. We will do so by using the resources of proof theory, in particular the sequent calculus, and by introducing the novel notion of *formal explanation* in first-order logic (i.e. we will extend and enrich the work developed in [2], [3]). The results we provide not only shed light on conceptual explanations themselves, but also on the role that logic and logical tools might play in the burgeoning field of inquiry concerning explanation.

[1] Lange, M., ***Because Without Cause: Non-causal Explanations in Science and Mathematics***, Oxford University Press, 2017.

[2] Poggiolesi, F., *On defining the notion of complete and immediate formal grounding*, ***Synthese***, vol. 193, pp. 3147-3167, 2016.

[3] Poggiolesi, F., *On constructing a logic for the notion of complete and immediate formal grounding*, ***Synthese***, vol. 195, pp. 1231-1254, 2018.

[4] Woodward, J., ***Making Things Happen: A Theory of Causal Explanation***, Oxford University Press, 2004.

▶ FRANCESCA POGGIOLESI, *Explanatory derivations in first-order logic.*
IHPST, UMR 8590,
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Zoltán Vidnyánszky – Finite and Infinite: an Interplay Between Distributed Computing and Borel Combinatorics. The field of Borel combinatorics investigates definable graphs on Polish spaces and aims at generalizing concepts of finite combinatorics to this realm. In the past couple of years a rich variety of connections have been found to the theory of distributed computing, in fact, it is often possible to prove transfer theorems between the two areas.

In my talk, I will survey some of these connections, focusing on the complexity related aspects of the two fields.

▸ ZOLTÁN VIDNYÁNSZKY, *Finite and Infinite: an Interplay Between Distributed Computing and Borel Combinatorics.*

Viorica Sofronie-Stokkermans – On symbol elimination in theory extensions and applications to parametric verification. We present a method proposed in [1] which, given a theory $\mathcal{T}_0$ allowing quantifier elimination, an extension $\mathcal{T}_0 \cup \mathcal{K}$ of $\mathcal{T}_0$ with additional function symbols Σ_1 whose properties are axiomatised by a set $\mathcal{K}$ of clauses, a set of parameters $\Sigma_{\mathsf{par}} \subseteq \Sigma_1$, and a set G of ground clauses, computes a universal formula Γ containing no functions in $\Sigma_1 \backslash \Sigma_{\mathsf{par}}$ with $\mathcal{T}_0 \cup \mathcal{K} \cup \Gamma \cup G \models \perp$ in a *hierarchical way*, relying on methods for quantifier elimination in $\mathcal{T}_0$. (If $\mathcal{T}_0$ does not allow quantifier elimination but has a model completion which does, we can use quantifier elimination in the model completion.)

We identify situations under which Γ is the *weakest* universal formula with the property above, and explain how we used this method for the verification of parametric systems: for generating (weakest) constraints on parameters under which certain properties are guaranteed to be inductive invariants [2]; for iteratively strengthening properties to obtain inductive invariants [3]; in problems from wireless research theory [4].

[1] Viorica Sofronie-Stokkermans, *On interpolation and symbol elimination in theory extensions*, ***Logical Methods in Computer Science***, vol. 14 (2018), no. 3.

[2] Viorica Sofronie-Stokkermans, *Parametric systems: Verification and synthesis*, ***Fundamenta Informaticae*** vol. 173 (2020), no. 2-3, pp. 91-138.

[3] Dennis Peuter, Viorica Sofronie-Stokkermans, *On invariant synthesis for parametric systems*, ***Automated Deduction - CADE-27 - 27th International Conference on Automated Deduction, Proceedings*** (Natal, Brazil), (Pascal Fontaine, editor), LNCS 11716, Springer, 2019, pp. 385–405.

[4] Dennis Peuter, Viorica Sofronie-Stokkermans, *Symbol elimination and applications to parametric entailment problems*, ***Frontiers of Combining Systems - 13th International Symposium, FroCoS-2021, Proceedings*** (Birmingham, UK) (Boris Konev and Giles Reger, editors), LNCS 12941, Springer, 2021, pp. 43–62.

▸ VIORICA SOFRONIE-STOKKERMANS, *On symbol elimination in theory extensions and applications to parametric verification.*
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Abstracts of the Tutorials

Logic Colloquium 2023

Contents

Floris van Doorn – Tutorial on interactive theorem proving in Lean. Do you want to try to prove some theorems in a computer proof assistant? Or do you want to learn the latest version of the Lean Theorem Prover? You will be able to do this in this tutorial. Please bring your laptop.

Lean is an interactive theorem prover that can be used to verify results in modern mathematics, such as a recent result by the Field's medalist Peter Scholze in condensed mathematics. Lean has a rapidly growing library of formalized mathematics, containing most of the material found in a typical undergraduate curriculum and various more advanced topics. Recently a new version of Lean has been released. This version, called Lean 4, besides being a proof assistant is a fully-fledged programming language, and most of the code for Lean 4 itself was developed in Lean 4.

In this tutorial I will be giving an hands-on introduction to using Lean. During most of the tutorial you will be proving results in Lean yourself, using a series or carefully chosen exercises to learn how to formalize proofs quickly. We will be using the online platform Gitpod to run Lean in the cloud, so that you don't have to install anything and it will run smoothly on less powerful machines.

► FLORIS VAN DOORN, *Tutorial on interactive theorem proving in Lean*.
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Itay Kaplan – Tutorial on Machine learning and model theory. I will give an overview of some connections between certain concepts in the theory of machine learning and notions in model theory. These connections were discovered and studied by many people in recent years, and lead to ideas and surprising results going both directions (from model theory to machine learning and vice-versa).

I will not assume any knowledge in model theory or machine learning.

▸ ITAY KAPLAN, *Tutorial on Machine learning and model theory.*

Abstracts of the Special Session Talks

Logic Colloquium 2023

Contents

APPLIED PROOF THEORY.

Horaţiu Cheval – Proof mining, applications to optimization, and interactive theorem proving. The research program of proof mining [3] is concerned with analyzing non-constructive mathematical proofs in order to extract additional quantitative (like effective bounds) or qualitative information (like the uniformity of the bounds or the weakening of the premises) from them. The analysis is guided by proof-theoretical instruments like Gödel's functional interpretation and Kohlenbach's monotone version thereof. In this way, theoretical guarantees on the extractability of such information can be given, in the form of general logical metatheorems.

We will begin by giving a brief introduction into the logical machinery behind proof mining. Then, we will present some new results in optimization and nonlinear analysis obtained in this context, concerning modified and generalized versions of the well-established Mann and Halpern iterations. These are joint work with Ulrich Kohlenbach and Laurenţiu Leuştean and can be found in [1, 2].

Finally, we will discuss the formalization of some of these results in the Lean theorem prover and our progress towards implementing the aforementioned logical instruments and metatheorems in Lean.

[1] Horaţiu Cheval and Laurenţiu Leuştean, *Quadratic rates of asymptotic regularity for the Tikhonov-Mann iteration*, ***Optimization Methods and Software***, vol. 37 (2022), no. 6, pp. 2225–2240.

[2] Horaţiu Cheval, Ulrich Kohlenbach and Laurenţiu Leuştean, *On modified Halpern and Tikhonov-Mann iterations*, ***Journal of Optimization Theory and Applications***, to appear.

[3] Ulrich Kohlenbach, ***Applied proof theory. Proof interpretations and their use in mathematics***, Springer Monographs in Mathematics, Springer-Verlag, 2008.

► HORAŢIU CHEVAL, *Proof mining, applications to optimization, and interactive theorem proving.*
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Morenikeji Neri – A metastable Kronecker's lemma with applications to the large deviations in the strong law of large numbers. Let $x_1, x_2, ...$ be a sequence of real numbers such that $\sum_{i=1}^{\infty} x_i < \infty$ and let $0 < a_1 \leq a_2 \leq ...$ be such that $a_n \to \infty$. Kronecker's lemma states,

$$\frac{1}{a_n} \sum_{i=1}^{n} a_i x_i \to 0$$

as $n \to \infty$

By applying Godel's Dialectica interpretation, we obtain a finitization of this result as well as the quantitative content of the classical proof of this result in the form of a rate of metastability.

We are then able to use our quantitative results to obtain new rates for the convergence in the strong law of large numbers, for both totally independent (a classic result of Kolmagorov) and pairwise independent sequences of random variables whose distributions are not assumed to be identical, thus, contributing to the study of large deviations in the strong law of large numbers. Furthermore, we are able to better existing rates found in [2].

Lastly, we present a contribution to computability theory, by constructing a sequence of rational numbers that satisfy the premise of Kronecker's lemma but do not converge with a computable rate of convergence (similar to the famous construction of Specker [4]). Thus, we are able to demonstrate the ineffectiveness of Kronecker's lemma. We then show how this ineffectiveness trickles down to the law of large numbers by constructing a sequence of computable random variables, that satisfy the premise of the laws of large numbers we shall study, whose averages do not converge with computable rates.

Our work can be seen as a contribution to the proof mining program, which aims to give a computational interpretation to prima facie non-effective proofs through the application of tools from logic. Our work builds on the new and exciting work on proof mining in probability/measure theory, in particular, [1] and [3].

[1] J. Avigad and P. Gerhardy and H. towsner, *Local stability of ergodic averages*, ***Transactions of the American Mathematical Society***, vol. 362, no. 1, pp. 261–288.

[2] N. LUZIA, *A simple proof of the strong law of large numbers with rates*, ***Bulletin of the Australian Mathematical Society***, vol. 97, no. 3, pp. 513–517.

[3] J. Avigad and E. Dean and J. Rute, *A metastable dominated convergence theorem*, ***Journal of Logic and Analysis***, vol. 4, pp. 3–19.

[4] E. Specker, *Nicht Konstruktiv Beweisbare Sätze der Analysis*, ***Journal of Symbolic Logic***, vol. 14, no. 3, pp. 145–158.

▸ MORENIKEJI NERI, *A metastable Kronecker's lemma with applications to the large deviations in the strong law of large numbers.*
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Nicholas Pischke – Intensional Methods in Applied Proof Theory. The logical substrate of applied proof theory in its modern form are the so-called logical metatheorems that classify (and allow for the extraction of) the computational content of mathematical theorems in various areas of pure and applied mathematics (see [3, 4, 5]). In the context of such metatheorems, one (necessarily) has to restrict the extensionality principles allowed in the underlying formal systems (i.e. in general only a weak rule of extensionality will be allowed, see the discussion in [5]). This normally poses only minor problems in actual applications as most objects treated by these systems are naturally extensional. In recent work however, areas of applications have emerged where extensionality issues already occur at the level of the definitions of the most basic objects. I will discuss some recent approaches to logical metatheorems and their underlying systems which crucially rely on the use of intensional objects to avoid such extensionality issues. In particular, I will illustrate the versatile applicability of such intensional approaches by discussing their use in treating set-valued operators [2] (with the prominent examples of accretive and monotone operators in Banach and Hilbert spaces, respectively) as well as dual spaces for general normed spaces and notions from convex analysis like gradients and conjugate functions [1].

[1] N. Pischke, *Proof mining for the dual of a Banach space with extensions for Fréchet-differentiable functions*, 2023, 24 pages. Manuscript in preparation.

[2] N. Pischke, *Logical metatheorems for accretive and (generalized) monotone set-valued operators*, 2022, 38 pages. Available at `https://arxiv.org/abs/2205.01788`.

[3] P. Gerhardy and U. Kohlenbach, *General logical metatheorems for functional analysis*, ***Transactions of the American Mathematical Society***, vol. 360 (2008), pp. 2615–2660.

[4] U. Kohlenbach, *Some logical metatheorems with applications in functional analysis*, ***Transactions of the American Mathematical Society***, vol. 357 (2005), no. 1, pp. 89–128.

[5] U. Kohlenbach, ***Applied Proof Theory: Proof Interpretations and their Use in Mathematics***, Springer Monographs in Mathematics, Springer-Verlag Berlin Heidelberg, 2008.

► NICHOLAS PISCHKE, *Intensional Methods in Applied Proof Theory.*
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COMPUTABILITY.

Elvira Mayordomo – Extensions of the point to set principle. Effective and resource-bounded dimensions were defined by Lutz in [5] and [4] and have proven to be useful and meaningful for quantitative analysis in the contexts of algorithmic randomness, computational complexity and fractal geometry (see the surveys [1, 6, 2, 12] and all the references in them).

The point-to-set principle (PSP) of J. Lutz and N. Lutz [8] fully characterizes Hausdorff and packing dimensions in terms of effective dimensions in the Euclidean space, enabling effective dimensions to be used to answer open questions about fractal geometry, with already an interesting list of geometric measure theory results (see [3, 11] and more recent results in [7, 14, 16, 15]).

In this talk I will review the point-to-set principles focusing on its recent extensions to separable spaces [9] and to Finite-State dimensions [13], and presenting open questions on the oracle and oracle access in PSP.

[1] R. G. Downey and D. R. Hirschfeldt. *Algorithmic randomness and complexity*. Springer-Verlag, 2010.

[2] J. M. Hitchcock, J. H. Lutz, and E. Mayordomo. The fractal geometry of complexity classes. *SIGACT News Complexity Theory Column*, 36:24–38, 2005.

[3] J. Lutz and N. Lutz. Who asked us? how the theory of computing answers questions about analysis. In *Complexity and Approximation: In Memory of Ker-I Ko*. Springer, ding-zhu du and jie wang (eds.) edition, 2020.

[4] J. H. Lutz. Dimension in complexity classes. *SIAM Journal on Computing*, 32(5):1236–1259, 2003.

[5] J. H. Lutz. The dimensions of individual strings and sequences. *Information and Computation*, 187(1):49–79, 2003.

[6] J. H. Lutz. Effective fractal dimensions. *Mathematical Logic Quarterly*, 51(1):62–72, 2005.

[7] J. H. Lutz. The point-to-set principle, the continuum hypothesis, and the dimensions of hamel bases. *Computability*, 2022. To appear.

[8] J. H. Lutz and N. Lutz. Algorithmic information, plane Kakeya sets, and conditional dimension. *ACM Transactions on Computation Theory*, 10, 2018. Article 7.

[9] J. H. Lutz, N. Lutz, and E. Mayordomo. Dimension and the structure of complexity classes. *Theory of Computing Systems*. To appear.

[10] J. H. Lutz, N. Lutz, and E. Mayordomo. Dimension and the structure of complexity classes. Technical Report arxiv.org:2109.05956, arxiv.org, 2021.

[11] J. H. Lutz and E. Mayordomo. Algorithmic fractal dimensions in geometric measure theory. In V. Brattka and P. Hertling, editors, *Handbook of Computability and Complexity in Analysis*. Springer-Verlag, 2021.

[12] E. Mayordomo. Effective fractal dimension in algorithmic information theory. In *New Computational Paradigms: Changing Conceptions of What is Computable*, pages 259–285. Springer-Verlag, 2008.

[13] E. Mayordomo. A point to set principle for finite-state dimension. Technical Report arXiv:2208.00157, Arxiv, 2022.

[14] T. Slaman. On capacitability for co-analytic sets. *New Zealand Journal of Mathematics*, 52:865—-869, 2022.

[15] D. Stull. Optimal oracles for point-to-set principles. In P. Berenbrink and B. Monmege, editors, *39th International Symposium on Theoretical Aspects of Computer Science (STACS 2022)*, volume 219 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 57:1–57:17. Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2022.

[16] D. M. Stull. The dimension spectrum conjecture for planar lines. In M. Bojańczyk, E. Merelli, and D. P. Woodruff, editors, *49th International Colloquium*

on Automata, Languages, and Programming (ICALP 2022), volume 229 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 133:1–133:20, Dagstuhl, Germany, 2022. Schloss Dagstuhl – Leibniz-Zentrum für Informatik.

▸ ELVIRA MAYORDOMO, *Extensions of the point to set principle.*
Departamento de Informática e Ingeniería de Sistemas, Instituto de Investigación en Ingeniería de Aragón, Universidad de Zaragoza, Iowa State University.
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Keng Meng Ng – Classifications in effective topology and computable analysis. We discuss some recent work on effective topological spaces and some attempts to classify spaces using computablity notions. We discuss notions such as universality, metrisability and presentability from the effective point of view. We also discuss how to calibrate spaces via a degree theoretic approach.

▸ KENG MENG NG, *Classifications in effective topology and computable analysis.*

Manlio Valenti – On the structure of Weihrauch degrees. Despite recent efforts, there are still several unanswered questions about the algebraic structure of the Weihrauch lattice. In this talk, we will explore some of these questions. After a brief introduction on Weihrauch/Medvedev degrees, we will present some recent results about the existence of chains in the Weihrauch degrees and provide a characterization for when "long" chains have an upper bound. This is also related to the problem of determining the cofinality of the degrees. We will show that, while for the Medvedev degrees the existence of a cofinal chain is equivalent to CH, for the Weihrauch degrees it is provable in ZFC that there are no cofinal chains. Finally, we will discuss some results on the extendibility of antichains and provide some sufficient conditions for antichains to be extendible. All these results showcase how, despite the close interplay between Medvedev and Weihrauch reducibility, the two lattices have a very different structure.

This is joint work with Steffen Lempp and Alberto Marcone.

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LOGIC AND COMPUTATION.

Anupam Das – Fixed points and circularity in logic and computation. Classical approaches to logic and computation typically restrict induction and recursion principles, relating logical constraints to resource bounds. However such approaches offer only a 'black box' treatment induction and recursion, admitting no finer logical or computational decomposition. Contrast this with, say, the use of an ω-rule in proof theory, recovering a metalogical analysis at the cost of finite presentability. However there is another, perhaps more drastic, approach: circular reasoning. Here the dependency graph of a proof need not be well-founded but is typically regular, akin to low-level computational models with loops while nonetheless enjoying excellent metalogical properties. Logical soundness (or computational totality) is guaranteed by an external condition inspired by ω-automaton theory.

In this talk I will survey some recent advances at the interface of proof theory and computation via cyclic proofs. At one end, complexity theory, I will show how ideas from Implicit Computational Complexity induce natural combinatorial properties on non-wellfounded proofs that yield expressive characterisations of (non-uniform) complexity classes. At the other end, recursion theory, I will show how circular systems have allowed us to calibrate the computational expressivity of (co)recursion in typed programming languages.

This talk is based on the references below, several of which are joint work with Gianluca Curzi.

[1] Anupam Das, *A circular version of Gödel's T and its abstraction complexity*, ***arXiv***, vol. abs/2012.14421, https://arxiv.org/abs/2012.14421, 2020.

[2] ——— *On the logical strength of confluence and normalisation for cyclic proofs*, ***FSCD '21*** Buenos Aires, Argentina, Naoki Kobayashi, vol. 195, Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2021, pp. 29:1–29:23.

[3] Gianluca Curzi & Anupam Das, *Cyclic implicit complexity*, ***LICS '22*** (New York, NY, USA), ACM, 2022, pp. 19:1–19:13.

[4] ——— *Non-uniform complexity via non-wellfounded proofs*, ***CSL '23*** Warsaw, Poland, Bartek Klin and Elaine Pimentel, vol. 252, Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2023, pp. 16:1–16:18.

[5] ——— *Computational expressivity of (circular) proofs with fixed points*, ***arXiv*** (accepted to LICS '23), vol. abs/2302.14825, https://arxiv.org/abs/2302.14825, 2023.

▶ ANUPAM DAS, *Fixed points and circularity in logic and computation.*
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Ján Pich – Towards $\mathsf{P} \neq \mathsf{NP}$ from Extended Frege lower bounds. The *proof complexity* approach to the P versus NP problem, sometimes refereed to as the Cook-Reckhow program, proceeds by proving lower bounds on lengths of proofs of tautologies in increasingly powerful proof systems - $\mathsf{NP} \neq \mathsf{coNP}$ (and hence $\mathsf{P} \neq \mathsf{NP}$) if and only if all propositional proof systems have hard sequences of tautologies that require superpolynomial proof size. A problem with the approach is that we do not know if we ever reach the point of proving a superpolynomial lower bound for all proof systems, if we focus only on concrete ones. In particular, even if we prove lower bounds on lengths of proofs in strong propositional proof systems such as Extended Frege, we might not be able to conclude that $\mathsf{P} \neq \mathsf{NP}$. In this talk we will connect this issue to several classical questions in complexity theory such as the problem of basing the security of cryptography on $\mathsf{P} \neq \mathsf{NP}$.

The talk is based on joint work with Rahul Santhanam.

▸ JÁN PICH, *Towards* $\mathsf{P} \neq \mathsf{NP}$ *from Extended Frege lower bounds.*
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Robert Robere – On propositional proofs and total search problems. Recent work has illustrated a deep relationship between the theories of *propositional proof systems* and *total NP search problems* (TFNP). The basic correspondence allows us to associate a total search problem S with each propositional proof system P such that the following holds: for every tautology T, T has a short proof in P if and only if proving T can be "efficiently reduced" to proving the totality of S. This allows us to define a theory of reducibility for proof systems that is analogous to classical reducibility in complexity theory, and it has led to the resolution of a number of open problems in both proof complexity and the theory of TFNP. In this talk we will introduce and survey this recent work.

▸ ROBERT ROBERE, *On propositional proofs and total search problems*.
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LOGIC AND PHILOSOPHY.

Carolin Antos – Formal concepts, defectiveness and pluralism. Formal concepts seem to require a different treatment than concepts from the empirical sciences. It is often assumed that their stability and fixedness makes them impervious against problems occurring with concept change, defectiveness or concept pluralism. In this talk I show how defectiveness can occur in formal concepts without giving up the claim that they are stable and fixed. I will also show how this can lead to a form of concept pluralism in formal contexts.

▸ CAROLIN ANTOS, *Formal concepts, defectiveness and pluralism.*
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Luca Tranchini – Intensional aspects of proof-theoretic semantics. In spite of significant mathematical results, comparatively little attention has been devoted to the notion of identity of proofs in the philosophical literature. I will argue that this is unfortunate, as identity of proofs is a powerful tool to investigate intensional aspects of meaning, provided that meaning is characterized in terms of inference rules. To substantiate this claim I will summarize the results of two lines of research (pursued in joint work with Paolo Pistone and Peter Schoeder-Heister respectively) in which identity of proofs has been applied to the study of the notion of harmony and of paradoxes. If time permits, I will briefly discuss some open problems and questions of mathematical, historical and philosophical nature concerning identity of proofs.

▸ LUCA TRANCHINI, *Intensional aspects of proof-theoretic semantics.*

Jack Woods – Prospects for Modest Inferentialism. In this paper I will argue that the combination of two insights from Prior, (1) that there are tonkish sets of natural deduction rules, and (2) that model theoretic accounts of meaning should be viewed as models of underlying intuitive meaning, together show that an otherwise promising form of inferentialism will not work. In particular, I argue that there is no way to formulate conditions on the intuitive meaning of the connectives which simultaneously (a) allow the natural deduction rules to specify the correct particular model theoretic meanings of connectives like conjunction, negation, disjunction, and the conditional while (b) not tacitly specifying the meaning of some of these connectives independently of the natural deduction rules. The upshot of this is that modest forms of inferentialism are unworkable. This is no idle result; philosophers have been repeatedly tempted by this position, even if under slightly different guises.

► JACK WOODS, *Prospects for Modest Inferentialism.*
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MODEL THEORY.

Vahagn Aslanyan – The Existential Closedness with Derivatives conjecture for the j-function. I will discuss the Existential Closedness conjecture for the modular j-function together with its first two derivatives. It is about the solvability of systems of equations involving j, j', j'' in the complex numbers and is an analogue of Zilber's Exponential Closedness conjecture which is about solvability of equations involving complex exponentiation. I will then explain why these two conjectures are qualitatively different, and what current approaches and partial results are known. I will also show that the Existential Closedness with Derivatives conjecture is significantly harder than its version for j without derivatives. If time permits, I will say a few words about the Modular Zilber-Pink with Derivatives conjecture and how it is related to Existential Closedness with Derivatives.

▸ VAHAGN ASLANYAN, *The Existential Closedness with Derivatives conjecture for the j-function*.
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Ulla Karhumäki – Groups of finite Morley rank and supertight automorphisms. After Morley proved his celebrated Categoricity theorem in 1965, a certain notion of dimension—today called the *Morley rank*—was recognised fundamental in model theory. This notion generalises the algebraic *Zariski dimension* and thus the class of groups of finite Morley rank generalises the class of algebraic groups over algebraically closed fields. Another class of groups studied by model theorists is that of *pseudofinite* groups. These are groups whose first-order theory only contains sentences which hold in some finite group. It is known that infinite simple pseudofinite groups are (twisted) Chevalley groups over pseudofinite fields and it is *conjectured* that infinite simple groups of finite Morley rank are Chevalley groups over algebraically closed fields; this conjecture is called the *Cherlin-Zilber conjecture*.

In her PhD thesis, Uğurlu Kowalski captured the algebraic behaviour of an infinite power of the Frobenius automorphism into the notion of *supertight automorphism* and suggested a new approach towards the Cherlin-Zilber conjecture. She proved that an infinite simple group of finite Morley rank with a supertight automorphism whose fixed point subgroups are pseudofinite contains an infinite simple pseudofinite subgroup so that the definable closure of this subgroup is the ambient group of finite Morley rank. We will see that if the Lie rank of the simple pseudofinite subgroup is one, then the group of finite Morley rank is algebraic and that, under suitable assumptions, if the Lie rank of the simple pseudofinite subgroup is greater or equal to three, then the group of finite Morley rank is again algebraic. The former result is joint work with Pınar Uğurlu Kowalski and the latter result is joint work with Adrien Deloro and Pınar Uğurlu Kowalski.

▸ ULLA KARHUMÄKI, *Groups of finite Morley rank and supertight automorphisms*.
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Konstantinos Kartas – Beyond the Fontaine-Wintenberger theorem. The idea that p-adic fields are in many ways similar to power series fields over finite fields has been highly influential in arithmetic. This philosophy has had two formal justifications, which were also used to transfer certain results across the two worlds. On one hand, the classical work by Ax-Kochen/Ershov in the '60s achieves a transfer principle when $p \to \infty$. On the other hand, Scholze's recent theory of perfectoid spaces works for fixed p but in the presence of high (and wild) ramification. I will first review those two methods and then mention some recent joint work with Franziska Jahnke in which we use model-theoretic tools to uncover certain new phenomena in perfectoid arithmetic.

▶ KONSTANTINOS KARTAS, *Beyond the Fontaine-Wintenberger theorem.*
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SET THEORY.

Takehiko Gappo – Chang-type models of determinacy. In [1], a new model of the Axiom of Determinacy was introduced by Grigor Sargsyan. This model is "Chang-type," in the sense that it contains δ^ω for some ordinal $\delta > \Theta$. First we will present two recent results using such a Chang-type model of determinacy. One is the proof of determinacy in the Chang model from a hod mouse with a Woodin limit of Woodin cardinals, and the other is a consistency result on ω-strongly measurable cardinals in HOD. We will also introduce a Chang-type model of determinacy with supercompact measures, which extends the result of [1]. This talk is based on several joint works with Navin Aksornthong, James Holland, Sandra Müller, and Grigor Sargsyan.

[1] Grigor Sargsyan, *Covering with Chang models over derived models*, ***Advances in Mathematics***, vol. 384 (2021), no. 107717.

▸ TAKEHIKO GAPPO, *Chang-type models of determinacy*.
Institute of Discrete Mathematics and Geometry, TU Wien, Wiedner Hauptstrasse 8-10/104, 1040 Vienna, Austria..
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Andreas Lietz – Forcing "NS_{ω_1} is ω_1-dense" from large cardinals. An ideal I on ω_1 is ω_1-dense if $(\mathcal{P}(\omega_1) - I)/I$ with the order induced by inclusion has a dense subset of size ω_1. Using his $\mathbb{P}_{\max}$-variation $\mathbb{Q}_{\max}$, W. Hugh Woodin [1] has shown that ZFC+"NS_{ω_1} is ω_1-dense" holds in generic extensions of canonical determinacy models. Assuming there is an inaccessible cardinal κ which is a limit of $<\kappa$-supercompact cardinals, we show that there is a stationary set preserving forcing $\mathbb{P}$ so that

$$V^{\mathbb{P}} \models \text{"NS}_{\omega_1} \text{ is } \omega_1\text{-dense"}.$$

This answers a question of Woodin [1]. To do so, we introduce a forcing axiom QM and force it true from large cardinals using two new iteration theorems which allow for destroying stationary sets. We then prove that QM implies the $\mathbb{Q}_{\max}$-version of Woodin's $(*)$-axiom by modifying methods of Asperó–Schindler [2] from their proof of "MM^{++} implies $(*)$". Along the way we get a few other new implications of the form "MM^{++} implies $(*)$".

[1] W. Hugh Woodin, ***The Axiom of Determinacy, Forcing Axioms, and the Nonstationary Ideal***, De Gruyter Series in Logic and its Applications, Walter de Gruyter GmbH & Co. KG, Berlin, 2010.

[2] David Asperó and Ralf Schindler, *Martin's Maximum^{++} implies Woodin's axiom* $(*)$, ***Annals of Mathematics. Second Series***, vol. 193 (2021), no. 3, pp. 793–835.

► ANDREAS LIETZ, *Forcing "NS_{ω_1} is ω_1-dense" from large cardinals.*
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Zhixing You – How far is almost strong compactness from strong compactness. In the paper [1], Bagaria and Magidor introduced the notion of almost strong compactness. Here an uncountable cardinal κ is almost strongly compact iff for every set I, every κ-complete filter on I can be extended to a δ-complete ultrafilter on I for every uncountable $\delta < \kappa$. Boney and Brooke-Taylor asked whether the least almost strongly compact cardinal, say κ, is strongly compact. Goldberg [2] gives a positive answer for this question in the case SCH holds from below and κ has uncountable cofinality. In this talk, we will give a negative answer for the general case by answering a relavant question of Bagaria and Magidor. This is joint work with Jiachen Yuan, [3].

[1] Joan Bagaria, Menachem Magidor, *Group radicals and strongly compact cardinals*, ***Transactions of the American Mathematical Society***, vol. 366 (2014), no. 4, pp. 1857-1877.

[2] Gabriel Goldberg, *Some combinatorial properties of ultimate L and V*, ***arxiv preprint***, arxiv:2007.04812 (2020).

[3] Zhixing You, Jiachen Yuan, *How far is almost strongly compact cardinal from strongly compact cardinal*, ***arxiv preprint***, arXiv:2302.13171v1 (2023).

► ZHIXING YOU, *How far is almost strong compactness from strong compactness.*
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Abstracts of the Contributed Talks

Logic Colloquium 2023

▶ MARCO ABBADINI, IVAN DI LIBERTI, *Jónsson-Tarski duality beyond dimension* 0.

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In 1951, Jónsson and Tarski obtained a representation theorem of modal algebras. A modern formulation is that the category of modal algebras is dually equivalent to the category of coalgebras for the Vietoris endofunctor over Stone spaces.

In the past years there has been a growing interest in analogues of Jónsson-Tarski duality in the larger settings of compact Hausdorff spaces and of stably compact spaces [2, 4, 3]. Our main result is that the category of coalgebras for the Vietoris functor on compact Hausdorff spaces is dually equivalent to a variety of infinitary algebras. For this, instead of working with the set of clopen subsets of a Stone space, we work with the set of continuous $[0,1]$-valued functions on a compact Hausdorff space. We deliver analogous results for the upper, lower and convex Vietoris endofunctors on stably compact spaces.

This abstract is based on [1].

[1] Marco Abbadini and Ivan Di Liberti, *Duality for coalgebras for Vietoris and monadicity*, ***Preprint at arXiv:2302.09529***.

[2] G. Bezhanishvili, N. Bezhanishvili, and J. Harding, *Modal compact Hausdorff spaces*, ***Journal of Logic and Computation***, vol. 25 (2015), no. 1, pp. 1–35.

[3] G. Bezhanishvili, L. Carai, and P. J. Morandi, *Modal operators on rings of continuous functions*, ***The Journal of Symbolic Logic***, vol. 87 (2022), no. 4, pp. 1322–1348.

[4] D. Hofmann, R. Neves, and P. Nora, *Limits in categories of Vietoris coalgebras*, ***Mathematical Structures in Computer Science***, vol. 29 (2019), no. 4, pp. 552–587.

▸ TIN ADLEŠIĆ, AND VEDRAN ČAČIĆ, *Model constructions of NFU*.
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Theory NF, short for New Foundations, was first introduced by Quine in 1937 as the middle way between type theory and ZF. During the search for consistency proof for NF, Jensen weakened the theory in 1969 by allowing the existence of atoms (urelements), and thus proved the consistency of this new modified theory, abbreviated by NFU. Jensen's model construction for NFU is rather hard to follow, so in 1977 Boffa presented a simplified version of it. Moreover, Boffa also presented in 1988 a completely new way of model construction from ZF. The third construction uses ultraproducts, and it was introduced by Holmes. We will present all three constructions with an emphasis on construction using ultraproducts. We will simplify this construction and develop it in a more straightforward way.

[1] M. Boffa, *The Consistency Problem for NFU*, ***The Journal of Symbolic Logic***, 1977.

[2] M. Boffa, *ZFJ and the consistency problem for NF.The Consistency Problem for NFU*, ***Jahrbuch der Kurt Gödel Gesellschaft 1***, 1988.

[3] R. Holmes, *Strong Axioms of Infinity in NFU*, ***The Journal of Symbolic Logic***, 2001.

[4] R. B. Jensen, *On the Consistency of a Slight (?) Modification of Quine's" New Foundations*, ***In Hintikka (ed.), Words and objections: Essays on the Work of W. V. Quine.***, 1969.

▸ TIN ADLEŠIĆ, VEDRAN ČAČIĆ, AND MARKO DOKO, *CoqNFU: formalizing New Foundations (with urelements) in Coq.*
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Today, the uncontested winner of the search for the axiomatization of set theory is Zermelo–Fraenkel set theory (ZF), commonly augmented with the axiom of choice. As the years have gone by, ZF has established itself not only as the champion of axiomatic set theory, but also the "default" foundation for the entirety of mathematics. Alternative approaches to set theory, such as Quine's New Foundations (NF), have been left by the wayside.

However, the modern advances in computer-aided theorem proving have shaken up the way people look at the foundations of mathematics. Type theory arose as the main theory on top of which assisted theorem provers should be built. Today set theory (i.e., ZF) is used by people doing mathematics on paper (a.k.a. "mathematicians"), while type theory is used by those doing mathematics on computers (a.k.a. "computer scientists"). What is the reason for this split? The answer might be that the convoluted ways one needs to argue if something is a set in ZF make it difficult to offload such reasoning to a computer, while checking if something is well-typed is a perfect task for computers.

Can we bridge this divide? Perhaps by "redeeming" set theory in the eyes of computers? If we are willing to consider alternative approaches to axiomatization, the future looks very promising. To establish whether a class is a set, the theory of New Foundations (with urelements), NF(U), requires only a simple decidable syntactic check on the formula used for set comprehension! Such syntactic checks, while tedious and unintuitive for humans, are perfect for computers.

In this presentation, we will take a look at the ongoing effort of formalizing NFU in the Coq theorem prover, as a proof of concept of the feasibility of building computer-verified proofs based on set-theoretic principles.

▸ MELISSA ANTONELLI, UGO DAL LAGO, DAVIDE DAVOLI, ISABEL OITAVEM AND PAOLO PISTONE, *Enumerating Error Bounded Polytime Algorithms Through Arithmetical Theories.*
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Since its early days, computer science has profoundly benefitted from the several interactions with mathematical logic. Among these, there is the possibility of characterizing fundamental complexity classes within a purely logical framework [6, 8], thus considering them from a new viewpoint, less dependent on concrete machine models and explicit resource bounds. Yet, classes defined on the bases of *randomized* algorithms [10] have remained difficult to capture with the tools of logic and this is especially true for semantic classes, as **BPP**, which are, by their nature, more challenging to enumerate via recursion-theoretic means. Currently, it is simply not known whether an effective enumeration of the aforementioned probabilistic classes is possible. In fact, the definition of **BPP**, being based on error-bounded machines, is not amenable to an enumeration, but nobody knows if alternative presentations of this same class support it. Indeed, the sparse contributions along these lines are usually themselves *semantic* [5, 9] or restricted to deal with syntactic randomized classes, like **PP** [3, 4].

The present work makes a step towards a logical characterization of randomized classes by considering a language in which the probability of error can be kept under control *from within* the logic. Concretely, we introduce a minimal extension of the language of arithmetic, such that the bounded formulas provably total in a suitably-defined theory *á la Buss* [2, 7] (expressed in this new language) precisely capture polytime *random* functions. Then, we provide two characterizations of **BPP**, obtained by internalizing the error-bound check *within* a logical system: one relies on measure-sensitive quantifiers [1], the other expresses the "measurement" via standard first-order quantification. This leads us to the introduction of a family of effectively-enumerable subclasses of **BPP**, each consisting of languages captured by probabilistic Turing machines whose underlying error can be proved bounded in the corresponding arithmetical theory. As a paradigmatic consequence, we establish that polynomial identity testing is in $\mathbf{BPP}_{\mathsf{PA}}$.

[1] Melissa Antonelli, Ugo Dal Lago, Paolo Pistone, *On Measure Quantifiers in First-Order Arithmetic*, ***Connecting with Computability*** (Ghent, July 5-9 2021), (Lisbeth De Mol, Andreas Weiermann, Florin Manea, and David Fernández-Duque, editors), Springer, 2021, pp. 12–24.

[2] Samuel Buss, ***Bounded Arithmetic***, Princeton University, 1986.

[3] Ugo Dal Lago, Reinhard Kahle, and Isabel Oitavem, *A Recursion-Theoretic Characterization of the Probabilistic Class PP*, ***46th International Symposium on Mathematical Foundations of Computer Science (MFCS 2021)*** (Filippo Bonchi, and Simon J. Puglisi, editors), vol. 202, Leibniz International Proceedings in Informatics (LIPIcs), 2021, pp. 1–12.

[4] Ugo Dal Lago, Reinhard Kahle, and Isabel Oitavem, *Implicit Recursion-Theoretic Characterization of Counting Classes*, ***Archive for Mathematical Logic***, vol. 61 (2022), pp. 1129–1144.

[5] Ugo Dal Lago, and Paolo Parisen Toldin, *A Higher-Order Characterization of Probabilistic Polynomial Time*, ***Information and Computation***, vol. 241 (2015), pp. 114–141.

[6] Roland Fagin, *Generalized First-Order Spectra and Poynomial-Time Recognizable Sets*, ***Complexity of Computation*** (Richard Karp, editor), vol. 7, (1974), SIAM-AMS Proceedings, pp. 43–273.

[7] Fernando Ferreira, *Polynomial-Time Computable Arithmetic*, ***Logic and Computation*** (Wilfried Sieg, editors), vol. 106, AMS, 1990, pp. 137–158.

[8] Neil, Immerman, ***Descriptive Complexity***, Springer, 1999.

[9] Emil Jerábek, *Dual Weak Pigeonhole Principle, Boolean Complexity, and Derandomization*, ***Annals of Pure and Applied Logic***, vol. 129 (2004), no. 1, pp. 1–37.

[10] Rajeev Motwani and Prabhakar Raghavan, ***Randomized Algorithms***, Cambridge University Press, 1995.

▸ ALEKSI ANTTILA, MATILDA HÄGGBLOM, AND FAN YANG, *Axiomatizing modal inclusion logic.*
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We present a natural deduction axiomatization of modal inclusion logic. Modal inclusion logic is the modal variant of first-order inclusion logic, introduced by Galliani [1]. Like dependence logic [3] (Väänänen 2007), inclusion logic adopts team semantics, in which formulas are evaluated with respect to sets of evaluation points (teams) rather than single points. For modal logics with team semantics, teams are sets of possible worlds in a Kripke model. Modal inclusion logic is classical modal logic extended with inclusion atoms, which describe that the possible truth values a sequence of formulas can obtain in a team are also truth values another sequence of formulas has somewhere in the team.

We review the proof of Hella and Stumpf [2] that modal inclusion logic is expressively complete for classes of Kripke models with teams that are closed under unions, closed under k-bisimulation for some natural number k, and have the empty team property. Through the expressive completeness proof, we obtain a normal form for formulas in modal inclusion logic, which plays a central role in the proof of the completeness theorem. Our proof system for modal inclusion logic builds on the proof system defined for propositional inclusion logic by Yang [4].

[1] Pietro Galliani, *Inclusion and exclusion dependencies in team semantics - on some logics of imperfect information*, ***Annals of Pure and Applied Logic***, vol. 163 (2012), no. 1, pp. 68–84.

[2] Lauri Hella, Johanna Stumpf, *The expressive power of modal logic with inclusion atoms*, ***International Symposium on Games, Automata, Logics and Formal Verification*** vol. 6 (2015), pp. 129–143.

[3] Jouko Väänänen, ***Dependence logic: a new approach to independence friendly logic***, London Mathematical Society student texts, Cambridge University Press, 2007.

[4] Fan Yang, *Propositional union closed team logics*, ***Annals of Pure and Applied Logic***, vol. 173 (2022), no. 6, 103102.

▸ ZIBA ASSADI, *Decidability of the Multiplicative and Order Theory of Numbers*.
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(This is a joint work with SAEED SALEHI). The ordered structures of natural, integer, rational and real numbers are studied. The theories of these numbers in the language of order are decidable and finitely axiomatizable. Also, their theories in the language of order and addition are decidable and infinitely axiomatizable. For the language of order and multiplication, it is known that the theories of $\mathbb{N}$ and $\mathbb{Z}$ are not decidable (and so not axiomatizable by any computably enumerable set of sentences). By TARSKI's theorem, the multiplicative ordered structure of $\mathbb{R}$ is decidable also. Here, we prove this result directly by quantifier elimination and present an explicit infinite axiomatization. The structure of $\mathbb{Q}$ in the language of order and multiplication seemed to be missing in the literature. We showed the decidability of its theory by the technique of quantifier elimination; and by presenting an infinite axiomatization for this structure, we proved that it is not finitely axiomatizable.

[1] ASSADI, ZIBA & SALEHI, SAEED, *On Decidability and Axiomatizability of Some Ordered Structures*, ***Soft Computing***, vol. 23 (2019), no. 11, pp. 3615–3626.

[2] ASSADI, ZIBA, ***Decidability of the Multiplicative and Order Theory of Numbers***, Ph.D. thesis (2019), University of Tabriz (in Persian). English version (14 September 2020): arXiv:2009.06336.

▸ STEVE AWODEY, *Homotopy type theory: Ten years after.*
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In the 10 years since the IAS Program on Univalent Foundations, which culminated in the release of the HoTT Book [9], substantial progress has been made in the field of homotopy type theory on several fronts, including solutions to leading open problems with both logical and mathematical significance. In work by Coquand et al. [3], the simplicial model of univalence [4] was shown have a constructive counterpart, verifying Voevodsky's canonicity conjecture. A computational proof assistant [7] was engineered on this basis, and in 2022 was used to finally compute "Brunerie's number" [5], finishing the formal verification of a proof that was begun at the IAS of the calculation of the fourth homotopy group of the 3-sphere, $\pi_4(S^3)$ [2].

The homotopical semantics of Martin-Löf type theory originated with [1], and was conjectured at the time by the author to provide an internal logic for higher toposes [6]. This was established by Shulman [8] in 2019, giving semantics for HoTT in all Grothendieck ∞-toposes. This talk will report on current research relating the constructive models underlying the new generation of computational proof assistants with the classical homotopy theory of higher toposes.

[1] Steve Awodey and Michael Warren, *Homotopy theoretic models of identity types*, ***Mathematical Proceedings of the Cambridge Philosophical Society***, 2009.

[2] Guillaume Brunerie, *On the homotopy groups of spheres in homotopy type theory*, June 2016, (arXiv:1606.05916).

[3] Cyril Cohen, Thierry Coquand, Simon Huber, and Anders Mörtberg, *Cubical type theory: a constructive interpretation of the univalence axiom*, ***21st International Conference on Types for Proofs and Programs***, May 2015, Tallinn, Estonia, pp. 129–162.

[4] Chris Kapulkin and Peter LeFanu Lumsdaine, *The simplicial model of univalent foundations (after Voevodsky)*, ***Journal of the European Mathematical Society***, 2021.

[5] Axel Ljungström, *The Brunerie number is -2*, ***Homotopy Type Theory (online)***, June 2022, `homotopytypetheory.org/2022/06/09/the-brunerie-number-is-2`.

[6] Jacob Lurie, *Higher topos theory*, ***Princeton University Press***, 2009.

[7] Anders Mörtberg, *Cubical Agda*, ***Homotopy Type Theory (online)***, December 2018, `homotopytypetheory.org/2018/12/06/cubical-agda`.

[8] Michael Shulman, *All $(\infty, 1)$-toposes have strict univalent universes*, April 2019, (arXiv:1904.07004).

[9] The Univalent Foundations Program, ***Homotopy type theory: the univalent foundations of mathematics***, The Institute for Advanced Study, 2013, `homotopytypetheory.org/book`.

▶ GUILLERMO BADIA, RONALD FAGIN, AND CARLES NOGUERA, *Completeness theorems for first-order real-valued logics with multidimensional sentences.*
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Many-valued logics in general, and real-valued logics in particular, usually focus on a notion of consequence based on preservation of full truth, typically represented by the value 1 in the semantics given in the real unit interval $[0, 1]$. In a recent paper [1], Ronald Fagin, Ryan Riegel, and Alexander Gray have introduced a new paradigm that allows to deal with inferences in *propositional* real-valued logics based on a rich class of sentences, multi-dimensional sentences, that talk about combinations of any possible truth-values of real-valued formulas. They have given a sound and complete axiomatization that tells exactly when a collection of combinations of truth-values of formulas imply another combination of truth-values of formulas. In this talk, we will extend their work to the first-order (as well as modal) logic of multi-dimensional sentences. We will give axiomatic systems and prove corresponding completeness theorems, first assuming that the structures are defined over a fixed domain, and later for the logics of varying domains. As a by-product, we will also obtain a 0-1 law for finitely-valued versions of these logics.

[1] Ronald Fagin, Ryan Riegel, and Alexander Gray. *Foundations of Reasoning with Uncertainty via Real-valued Logics*, arXiv:2008.02429v2, 2021.

▸ GUILLERMO BADIA AND DAVID C. MAKINSON, *First-order friendliness*.
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The relation of *logical friendliness*, introduced in the propositional context in [1], has a very straightforward definition as a $\forall\exists$ version of the fundamental $\forall\forall$ notion of consequence. Specifically, if Γ is a set of formulae of classical propositional logic and ϕ is a formula of the same, Γ is said to be friendly to ϕ iff for every valuation v on the propositional variables occurring in formulae of Γ, if $v(\gamma) = 1$ for all $\gamma \in \Gamma$ then there is an extension of v to a valuation v' covering also any remaining variables in ϕ such that $v(\phi) = 1$. It is thus a weakening of classical consequence and if the existential quantifier in its definition is replaced by a universal one, it reverts to the classical relation.

So defined, friendliness has a number of interesting features. While lacking some familiar properties of classical consequence, it satisfies some others in full, as well as yielding 'local' versions of yet others, as shown in [1]. However, if we seek to extend the definition from the propositional to the first-order context, a number of options arise due to the greater complexity of the notion of a first-order model, with its ingredients of domain of discourse, values for individual constants, values for predicate and function letters, and the equality relation. The various options generate distinct relations, which differ in their behaviour. Indeed, two of the lessons of the present paper are that the concept of friendliness is less robust in the first-order context than in the propositional one and that even the seemingly best behaved of the possible first-order options is less regular than its propositional counterpart, notable with respect to compactness, axiomatizability and interpolation.

The full paper is available on arXiv at https://arxiv.org/abs/2210.13953.

[1] D. Makinson. Friendliness and sympathy in logic. In: JY Beziau (ed.) *Logica Universalis* (2nd Edition), pp. 195-224, Basel: Birkhauser Verlag, 2007. (Essentially the same material appeared under the title "Friendliness for logicians" in *We Will Show Them! Essays in Honour of Dov Gabbay*, vol 2, ed S. Artemov et. al, College Publications 2005, pp 259-292).

▸ PHILIPPE BALBIANI, TINKO TINCHEV, *Modal definability in Euclidean modal logics.*

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A Kripke frame (W, R) is called Euclidean if the accessibility relation R satisfies the condition: for all $s, t, u \in W$, if sRt and tRu then tRu and uRt. A modal logic L is called Euclidean if it is determined by a nonempty class of Euclidean frames, i.e. if L is an extension of the modal logic $K5$. For every logic L, let $Fr(L)$ be the class of all frames validating the theorems of L. A sentence A from the first-order language with equality and one binary predicate symbol is modally definable with respect to some class of frames if there is a modal formula φ from the classical propositional modal language such that A and φ are valid in the same frames from the class. Modal definability in a logic L problem asks whether there exists an algorithm that recognizes all modally definable with respect to $Fr(L)$ sentences. Correspondence problem in a logic L asks whether there exists an algorithm that for any sentence A and any modal formula φ recognizes whether A and φ are valid in the same frames from $Fr(L)$.

In this talk we present all Euclidean modal logics L such that the modal definability in L is decidable problem. We demonstrate also that these logics are exactly the Euclidean logics in which the correspondence problem is decidable.

▸ AINUR BASHEYEVA, SVETLANA LUTSAK, *On quasivarieties generated by some finite modular lattices.*
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In 1970 R. McKenzie proved that any finite lattice has a finite basis of identities. However the similar result for quasi-identities is not true. That is, there is a finite lattice that has no finite basis of quasi-identities (V.P. Belkin 1979). These results naturally arose the problem: Which finite lattices have finite bases of quasi-identities? The problem was suggested by V.A. Gorbunov and D.M. Smirnov in 1979. In 1984 V.I. Tumanov found sufficient condition consisting of two parts under which the locally finite quasivariety of modular lattices has no finite (independent) basis of quasi-identities. Also he conjectured that a finite (modular) lattice has a finite basis of quasi-identities if and only if a quasivariety generated by this lattice is a variety. In general, the conjecture is not valid. In 1989 W. Dziobiak found a finite lattice that generates finitely axiomatizable proper quasivariety. However the Tumanov's conjecture for modular lattices is still open.

The main goal of this work is to present a specific finite modular lattice such that the quasivariety generated by this lattice does not satisfy all conditions of Tumanov's theorem and has no finite basis of quasi-identities. The proof of this result gives many examples of finite lattices that confirm Tumanov's conjecture.

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▸ GIULIA BATTILOTTI, MILOŠ BOROZAN AND ROSAPIA LAURO GROTTO, *A hypothesis on the components of judgements following Freudian theory.*
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In his essay "On Aphasia" (1891) Freud proposed that words do not refer to objects, but to non-verbal open representations of objects operated by the mind, termed thing-presentations, which can access consciousness only when closed by words. Freud's approach is somewhat unique even in an epistemological perspective, and hence allows for addressing the issue of representation in new ways, in particular the issue of formal representation. We apply the approach to formal language, investigating its consequences in logic. The analysis is supported by further reading of Freudian theory in logical terms, developed by Freud himself, by Matte Blanco and Bion. We consider quantified formulae on non-extensional domains termed infinite singletons, that formalize thing-presentations of objects. They become closed predicates only when thing-presentations are closed by words. Modal assertions are amid open and closed presentations since the modal operator of S4 can attribute a stable value, even if undefined, to the object. In the model, it can be derived by abstracting with respect to collection of infinite singletons which refer to the same object. Assuming that such an abstraction can occur in the Unconscious, an infinite singleton would be characterized, for the collection of infinite singletons referring to the object. Assuming that some form of timing for the process of abstraction is recognized (due to the contact with reality), a sort of "priming" for that process can be considered. Then one can set different independent possibilities for priming, at the origin of our conscious judgements.

[1] Battilotti, G.; Borozan, M.; Lauro Grotto, R. Infinite Singletons and the Logic of Freudian Theory, Language and Psychoanalysis 10, 2021, 46–62.

[2] Battilotti, G.; Borozan, M.; Lauro Grotto, R. A modal interpretation of quantum spins and its application to Freudian theory. Entropy 24(10), 2022, 1419.

▸ GAIA BELARDINELLI, AND THOMAS BOLANDER, *Attention! Dynamic Epistemic Logic models of (in)attentive agents.*
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Attention is the crucial cognitive ability that limits and selects what information we observe. Previous work by Bolander et al. [1] proposes a model of attention based on dynamic epistemic logic (DEL) where agents are either fully attentive or not attentive at all. While introducing the realistic feature that inattentive agents believe nothing happens, the model does not represent the most essential aspect of attention: its selectivity. Here, we propose a generalization that allows for paying attention to subsets of atomic formulas. We introduce the corresponding logic for propositional attention, and show its axiomatization to be sound and complete. We then extend the framework to account for inattentive agents that, instead of assuming nothing happens, may default to a specific truth-value of what they failed to attend to (a sort of prior concerning the unattended atoms). This feature allows for a more cognitively plausible representation of the inattentional blindness phenomenon, where agents end up with false beliefs due to their failure to attend to conspicuous but unexpected events. We prove the extended logic to be sound and complete as well. Both versions of the model define attention-based learning through appropriate DEL event models based on a few and clear edge principles. While the size of such event models grow exponentially both with the number of agents and the number of atoms, we introduce a new logical language for describing event models syntactically and show that using this language our event models can be represented linearly in the number of agents and atoms. Furthermore, representing our event models using this language is achieved by a straightforward formalisation of the aforementioned edge principles.

The full paper is available here: https://arxiv.org/abs/2303.13494

[1] Thomas Bolander, Hans van Ditmarsch, Andreas Herzig, Emiliano Lorini, Pere Pardo, and François Schwarzentruber, *Announcements to Attentive Agents*, ***Journal of Logic, Language and Information***, vol. 25 (2016), pp. 1–35.

▸ LUCA BELLOTTI, *Notes on the (un)provability of consistency.*
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We present a few remarks on the classic problem of the unprovability of consistency of some important formal systems and on some ways to (partially) circumvent it which have been proposed. We consider the impossibility of certain constructive consistency proofs for set theory, the role of local set-theoretic reflection principles and of partial or indirect consistency statements for arithmetic, with a final remark on so-called Whiteley sentences.

► BRUNO BENTZEN, *Bishop's mathematical intuitionism*.
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In his seminal book *Foundations of Constructive Analysis*, Errett Bishop develops a form of constructive mathematics that rejects the theory of the continuum proposed by L.E.J. Brouwer and thus his intuitionistic mathematics altogether. While Bishop's brand of constructivism had considerably more success in drawing the attention of working mathematicians, discussions of the philosophy that underlies his program of constructivization of mathematics remain rare in the literature, with the exception of works by Nicolas Goodman, Helen Billinge, and Laura Crosilla.

In this talk I argue that despite his rejection of Brouwer's intuitionistic mathematics, the philosophical position that underlies Bishop's constructive mathematics can actually be identified as a form of mathematical intuitionism as well, broadly understood in Arend Heyting's sense as the commitment to the two following basic tenets:

- mathematics is meaningful and eludes formalization;
- all mathematical objects are mental constructions given in intuition.

I shall develop and defend an intuitionistic interpretation of Bishop's philosophical views by supporting the two tenets of intuitionism above with textual evidence from his philosophical writings. My intention is not to defend Bishop's own philosophical claims or suggest that he must have thought of himself as an intuitionist in my sense. All I am claiming is that his thought provides scope for a coherent intuitionist understanding of mathematics.

► GURAM BEZHANISHVILI, LUCA CARAI, AND PATRICK J. MORANDI, *Deriving Priestley and Esakia dualities and their generalizations from Pontryagin duality for semilattices.*

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Hofmann, Mislove, and Stralka [2] developed a version of Pontryagin duality that establishes a dual equivalence between the categories of semilattices and Stone semilattices. Since Stone semilattices are exactly algebraic lattices, it follows that the category of semilattices is equivalent to the category of algebraic lattices and maps that preserve arbitrary joins and compact elements. This equivalence on the object level is a reformulation of the well-known 1-1 correspondence between semilattices and algebraic lattices. The restriction to the distributive case yields an equivalence between the categories of distributive semilattices and algebraic frames.

By Priestley and Esakia dualities, the categories of distributive lattices and Heyting algebras are dual to the categories of Priestley and Esakia spaces, respectively. These dualities have been generalized to various subreducts of distributive lattices and Heyting algebras. We show how to derive Priestley and Esakia dualities and their generalizations from Pontryagin duality for semilattices. In particular, we show how to obtain the dualities for distributive and implicative semilattices developed by Bezhanishvili and Jansana [1]. This provides a frame-theoretic perspective on Priestley and Esakia dualities and their generalizations.

[1] G. Bezhanishvili and R. Jansana, *Duality for distributive and implicative semi-lattices*, 2008. Available from http://www.ub.edu/grlnc/docs/BeJa08-m.pdf.

[2] K. H. Hofmann, M. Mislove, and A. Stralka, ***The Pontryagin duality of compact* O*-dimensional semilattices and its applications***, Lecture Notes in Mathematics, Vol. 396, Springer-Verlag, 1974.

▸ NEER BHARDWAJ, GAL BINYAMINI, *Approximate Pila-Wilkie type counting for complex analytic sets*.

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Habegger [1] developed a variation of the Pila-Wilkie counting theorem, wherein they bound the number of rational points of bounded height that approximate, in a specific sense, sets definable in a polynomially bounded o-minimal structure. Working with this theme, we focus on bounded complex analytic sets, but improve on the relevant *Pila-Wilkie constant* so that it is, in a precise sense, more universal and does not depend as such on the specific set. Our tools involve, in particular, basic Nevanlinna theory; and our new feature entails applications such as a *transcendence measure* increment to Pila's Ax-Lindemann-Weierstrass argument, and certain computability results with regards to the Pila-Wilkie type constants involved. This is joint work with Gal Binyamini.

[1] Philipp Habegger, *Diophantine approximations on definable sets*, ***Selecta Mathematica. New Series***, vol. 24 (2018), no. 2, pp. 1633–1675.

▶ JOSÉ MIGUEL BLANCO, FÉLIX CUADRADO, *Formal modelling of distributed temporal graphs algorithms: the case of Raphtory.*

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The rise of temporal graphs has also produced many tools to delve into their analytics and provide real-time support for decision-making. Nevertheless, these tools are based on complex underlying models whose behaviour needs to be ensured so no unexpected side-effects or ill behaviour happens. For that matter formal modelling is an option that has been used extensively, ensuring results like the decidability of the system or enabling the possibility of performing a model check.

Thus, the main aim of this communication is to provide the formal modelling of Raphtory, an open-source platform for distributed real-time temporal graph analytics [1]. For that matter we will make use of Routley-Meyer semantics [2] as they are one of the best tools to model distributed systems, and we will extend them by introducing a time-flow with branching time operators. All this will allow us to obtain a perfect representation of Raphtory and derive its properties.

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[1] Benjamin Steer, Félix Cuadrado, Richard Clegg, *Raphtory: Streaming analysis of distributed temporal graphs*, ***Future Generation Computer Systems***, vol. 102 (2020), pp. 453–464.

[2] Richard Routley, Robert K. Meyer, Ross T. Brady, Valerie Plumwood, ***Relevant Logics and Their Rivals, vol. 1***, Ridgeview Publishing Company, 1982.

▸ S. BONZIO, V. FANO, P. GRAZIANI AND M. PRA BALDI, *A logical modeling of severe ignorance via Bochvar external logic.*
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The study of ignorance is certainly as old as the study of knowledge; however the formal study of the logic of ignorance is still a young area of research. In the epistemological studies of ignorance the standard view is to define it as lack of knowledge (see for example [8], [5], [7], [6]). We believe that this is the reason why also the formal study of the logic of ignorance has been developed with reference to the formal study of the logic of knowledge. This tradition is mainly due to the work of Hintikka [4], who distinguishes two notions of lack of knowledge relative to an agent, namely "a (an agent) does not know that φ" ($\neg\mathbf{K}_a\varphi$) and "a does not know whether φ" ($\neg\mathbf{K}_a\varphi \wedge \neg\mathbf{K}_a\neg\varphi$). It seems that, according to Hintikka, only the latter explicates the notion of ignorance; indeed, he [4, p.12] formalizes ignorance (of an agent a) as $\neg\mathbf{K}_a\varphi \wedge \neg\mathbf{K}_a\neg\varphi$. Such regimentation has become standard in the logical literature on ignorance. For this reason, by the expression "ignorance as lack/absence of knowledge", we will refer to Hintikka's view throughout this talk.

However, in more recent times, van der Hoek and Lomuscio [11] introduced a modal logic (**Ig**) where ignorance is modeled by a primitive modal operator, unrelated to (lack of) knowledge. The spirit behind **Ig** is expressing "ignorance as a first class citizen" [11, p.3]. However, despite their intention, their solution does not seem too far from Hintikka's lack of knowledge. The semantics of **I**, as we will show, is the same as in Hintikka [4, p.12], with the only difference that **Ig** "can not speak" about knowledge. Similarly, the *Logic of Unknown Truths* (LUT) and the subsequent logics of ignorance proposed by Steinsvold [10] subordinate the concept of ignorance to that of knowledge. In these logics the black box ($\blacksquare$) in fact stands for $\varphi \wedge \neg\mathbf{K}\varphi$; if the latter formula is true, and $\varphi \to \neg\mathbf{K}\neg\varphi$ holds, then also $\neg\mathbf{K}\varphi \wedge \neg\mathbf{K}\neg\varphi$ holds, which is again Hintikka's definition of ignorance.

Following the research trend opened in Fano and Graziani [2], this article intends to discuss the fact that *lack of knowledge* is just one way to look at ignorance and, taking up van der Hoek and Lomuscio's challenge, to introduce a logic which addresses the purpose of defining "ignorance as a first class citizen". In this paper, after discussing the consequences of defining ignorance as lack of knowledge (in the epistemic logic S_4), we introduce and investigate a modal logic having a primitive epistemic operator **I**, modeling ignorance. In particular, the idea we have in mind is that of modelling a type of *content-theoretic ignorance*, so to say an ignorance of something that stems from an unfamiliarity with its meaning, i.e. a *severe* notion of ignorance that implies a lack of awareness with respect to a subject-matter. In our view, this type of ignorance constantly affects the practice of science.

To achieve the goal of modeling severe ignorance, we base the semantics of our (modal) logic on the presence of a third truth-value, whose behaviour is infectious: we opt for Bochvar external logic, originally introduced in [1]. Our modal logic **SI** consists of a linguistic extension of Bochvar (external) logic via a primitive modality **I**, for ignorance.

The talk is organized into four parts: in the first part we introduce the standard (logical) approach to ignorance as "lack of knowledge". In the second part, we introduce Bochvar external logic. In particular, we extend the known results from [3], proving that Bochvar external logic is algebraizable with the quasivariety of Bochvar algebras as its equivalent algebraic semantics. In the third, it is introduced the logic **SI** of severe ignorance, its axiomatization for which we prove completeness with respect to a relational semantics and decidability (the proof follows the ideas of the modal logics based on weak Kleene logics introduced by Segerberg [9]). Finally, we conclude the talk with some remarks on the validity of certain formulas relevant to capture a severe notion of ignorance, and compare the differences between the standard view and the proposed logic for severe ignorance.

[1] D. Bochvar and M. Bergmann. On a three-valued logical calculus and its application to the analysis of the paradoxes of the classical extended functional calculus. *History and Philosophy of Logic*, 2(1-2):87–112, 1981. Translation of the original in Russian (Mathematicheskii Sbornik, 1938).

[2] V. Fano and P. Graziani. A working hypothesis for the logic of radical ignorance. *Synthese*, 199:601–616, 2021.

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[4] J. Hintikka. *Knowledge and Belief*. Ithaca: Cornell University Press, 1962.

[5] P. LeMorvan. On Ignorance: A reply to Peels. *Philosophia*, 39:335–344, 2011.

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[9] K. Segerberg. Some modal logics based on a three-valued logic. *Theoria*, 33(1):53–71, 1967.

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▶ S. BONZIO AND M. PRA BALDI, *On the structure of Bochvar algebras.*
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The recent years have seen a renaissence of interests and studies around weak Kleene logics, logical formalisms that were considered, in the past, perhaps less attractive in the panorama of three-valued logics, due to infectious behavior of the third-value. This late (re)discovery regards, almost exclusively, *internal* rather than *external* logics. The latter, in essence consisting of linguistic expansions of the former, includes Bochvar external logic (introduced in [1]) and the external version of Paraconsistent weak Kleene (introduced by Segerberg [11]). Bochvar external logic B_e is defined in the algebraic language $\mathcal{L}$: $\langle \neg, \vee, \wedge, J_0, J_1, J_2, 0, 1\rangle$ (of type $(1,2,2,1,1,1,0,0)$), as the logic induced by the single matrix $\langle \mathbf{WK}^e, \{1\}\rangle$ (whose algebraic reduct is recalled in Fig. 1).

	$\neg$
1	0
1/2	1/2
0	1

$\vee$	0	12	1
0	0	1/2	1
1/2	1/2	1/2	1/2
1	1	1/2	1

$\wedge$	0	1/2	1
0	0	1/2	0
1/2	1/2	1/2	1/2
1	0	1/2	1

φ	$J_0\varphi$
1	0
1/2	0
0	1

φ	$J_1\varphi$
1	0
1/2	1
0	0

φ	$J_2\varphi$
1	1
1/2	0
0	0

FIGURE 1. The algebra $\mathbf{WK}^e$.

Finn and Grigolia [4, 5] introduced a Hilbert-style axiomatization for B_e and also the class of Bochvar algebras.

DEFINITION 1. A Bochvar algebra $\mathbf{A} = \langle A, \vee, \wedge, \neg, J_0, J_1, J_2, 0, 1\rangle$ is an algebra of type $\langle 2,2,1,1,1,1,0,0\rangle$ satisfying the following identities and quasi-identities:

1. $x \vee x \approx x$;
2. $x \vee y \approx y \vee x$;
3. $(x \vee y) \vee z \approx x \vee (y \vee z)$;
4. $x \wedge (y \vee z) \approx (x \wedge y) \vee (x \wedge z)$;
5. $\neg(\neg x) \approx x$;
6. $\neg 1 \approx 0$;
7. $\neg(x \vee y) \approx \neg x \wedge \neg y$;
8. $0 \vee x \approx x$;
9. $J_2 J_k x \approx J_k x$, for every $k \in \{0,1,2\}$;
10. $J_0 J_k x \approx \neg J_k x$, for every $k \in \{0,1,2\}$;
11. $J_1 J_k x \approx 0$, for every $k \in \{0,1,2\}$;
12. $J_k(\neg x) \approx J_{2-k} x$, for every $k \in \{0,1,2\}$;
13. $J_i x \approx \neg(J_j x \vee J_k x)$, for $i \neq j \neq k \neq i$;
14. $J_k x \vee \neg J_k x \approx 1$, for every $k \in \{0,1,2\}$;

15. $((J_i x \vee J_k x) \wedge J_i x) \approx J_i x$, for $i, k \in \{0, 1, 2\}$;
16. $x \vee J_k x \approx x$, for $k \in \{1, 2\}$;
17. $J_0(x \vee y) \approx J_0 x \wedge J_0 y$;
18. $J_2(x \vee y) \approx (J_2 x \wedge J_2 y) \vee (J_2 x \wedge J_2 \neg y) \vee (J_2 \neg x \wedge J_2 y)$;
19. $J_0 x \approx J_0 y \;\&\; J_1 x \approx J_1 y \;\&\; J_2 x \approx J_2 y \;\Rightarrow\; x \approx y$.

The class $\mathcal{BCA}$ of Bochvar algebras forms a proper quasi-variety. Only recently [2], it has been shown that $\mathcal{BCA}$ algebrizes Bochvar external logic.

In this contribution, we show that the algebraic construction of the Płonka sum [7, 8] (more comprehensive expositions are [9], [3, Ch. 2]) allows to provide a representation theorem for Bochvar algebras. Such a representation allows us to characterize logical filters and to provide a constructive proof of the that the $\mathcal{BCA}$ is generated by the single algebra $\mathbf{WK}^e$.[1]Moreover, we describe the lattice of (non-trivial) subquasivarieties of Bochvar algebras and, dually, the lattice of extensions of B_e, which consist of a three-elements chain, and prove that B_e is not passively structurally complete, while its non-trivial extension – here named NB_e – is structurally complete (see [6] and [10]). In the final part of the talk, we will focus on relevant algebraic properties for algebraizable logics and, upon relying once more on the Płonka sum representation theorem, we show that $\mathcal{BCA}$ has surjective epimorphisms and the amalgamation property.

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▸ SAMUEL BRAUNFELD, ANUJ DAWAR, IOANNIS ELEFTHERIADIS, AND ARIS PAPADOPOULOS,

Monadic NIP in monotone classes of relational structures.

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We prove that for any monotone class of finite relational structures, the first-order theory of the class is NIP in the sense of stability theory if, and only if, the collection of Gaifman graphs of structures in this class is nowhere dense. This generalises to relational structures a result previously known for graphs and answers an open question posed by Adler and Adler in [1].

The result is established by the application of Ramsey-theoretic techniques and shows that the property of being NIP is highly robust for monotone classes. We also show that the model-checking problem for first-order logic is intractable on any class of monotone structures that is not (monadically) NIP. This is a contribution towards the conjecture of Bonnet et al. from [2] that the hereditary classes of structures admitting fixed-parameter tractable model-checking are precisely those that are monadically NIP.

[1] Hans Adler and Isolde Adler, *Interpreting nowhere dense graph classes as a classical notion of model theory*, ***European Journal of Combinatorics***, vol. 36 (2014), pp. 322–330.

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▸ SAMUEL BRAUNFELD, JAROSLAV NEŠETŘIL, PATRICE OSSONA DE MENDEZ, AND SEBASTIAN SIEBERTZ, *Low covers of graph classes preserve stability and NIP*.
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This talk will describe a connection between model theory and structural graph theory. The *low covers* construction allows for describing graphs from a given class as locally looking like graphs from a simpler class, and has been a key tool in the theory of sparse graph classes. We describe how the model-theoretic properties of stability and NIP are preserved by this construction, thus lifting local structure to global structure. This opens the way to extending techniques from graph sparsity theory to analyze model-theoretically tame graph classes.

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▸ SELMER BRINGSJORD, NAVEEN SUNDAR GOVINDARAJULU, ALEXANDER BRINGSJORD,

Three-dimensional hypergraphical natural deduction.

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As is widely known, natural deduction was first presented in 1934, independently by Gentzen [2] and Jáśkowski [3]; this event gave rise to three fundamental and fundamentally different ways of rendering such deduction precise, a trio that firmly persists to the present day. Gentzen gave a tree format for natural deduction; Jáśkowski gave a box-based one, and a tabular, "bookkeeping" one. (Pelletier [4] credits Suppes with a fourth way, but this is controversial, since Suppes' innovation is a formalism for tracking suppositions that remain in force as a proof proceeds in Jáśkowskian tabular fashion.) We first briefly review the three natural-deduction ways, and show our expansion of Genzen's trees into a novel system based on (usually directed, acyclic) *hypergraphs*. (Hypergraphs are covered e.g. in [1] and — more recently — in [5].) We next show that our system (in two-dimensional mode) is implemented and integrated with automated reasoners (= "oracles"), via specimen formal proofs that range over third-order logic, with additional optional modal operators available for the alethic, epistemic, deontic cases etc. We then explain that the three original specifications for natural deduction, despite their differences, are most assuredly in any case two-dimensional: each element therein is located somewhere in a backdrop of an x and a y axis, as in simple, discrete Euclidean two-space. We then reveal how natural deduction in our hypergraphical environment can be better expressed in *three-dimensional* hypergraphs. Our 3D hypergraphical proofs use a third z axis on which formulae in nodes can be located, to and from which run inferential arcs. This third dimension, as we explain and show in relevant proofs, can be interpreted, within proof-theoretic semantics, as e.g. determining the degree of "prominence" of formulae and inferential links in a given proof.

We conclude with some remarks about connections we perceive between 3D hypergraphical natural reasoning and the dream of Leibniz to find a rigorous universal reasoning system. Leibniz dreamed of an interoperating pair: (i) the *calculus ratiocinator*, the machine or mechanical system, which brings information expressed in (ii) the universal rational calculus, or *characteristica universalis*, to life. Our system, we claim, realizes this dream.

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[3] Stanislaw Jáśkowski, *On the Rules of Suppositions in Formal Logic*, ***Studia Logica***, vol. 1 (1934), no. 1, pp. 5–32.

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▸ PIETRO BROCCI, *Disquotation, minimality and proof-theoretic power*.
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The popularization of the deflationist doctrine on truth has brought new attention to disquotational principles, i.e. $T(A) \leftrightarrow A$. The reason for this is that, according to deflationists, truth is merely a logico-mathematical device and it should be logically represented by principles that express its function. Since disquotational principles are generally successful in doing so, disquotational theories of truth are being investigated now more than ever.

The aim of the paper is to assess the status and logical force of disquotational principles in the framework of axiomatic theories of truth, in particular for theories formulated in classical logic. First, we survey the theories formulated by Halbach [3], Schindler [5] and Picollo [4]. Picollo's theory achieves proof-theoretic strength by changing the base theory, making it not comparable with the others. Schindler's proposal achieves the strongest proof-theoretic power but, we argue, its axioms are not justified independently of its strength. Thus, Halbach's theory, PUTB, is still the best candidate for an axiomatization of disquotational principles in classical logic, being well-motivated and proof-theoretically as strong as the theory that states the existence of fixed-points for arbitrary positive inductive definitions, $\widehat{\mathsf{ID}}_1$.

In the second part of the paper, we show that PUTB is capable of capturing even more mathematical reasoning. To do so, we extend it by means a minimality principle in the style of Burgess' theory in [1]. We prove that this new theory, PUTB_μ, is proof-theoretically as strong as ID_1, i.e. the theory that states the existence of *minimal* fixed-points for arbitrary positive inductive definitions. This makes PUTB_μ as strong as Burgess' theory, KFB. Fujimoto in [2] argues that proof-theoretic equivalence results are not enough to show that two theories capture the same concept of truth, for this purpose they introduce the notion of relative truth definability. Therefore, we conclude by proving that KFB is relatively truth definable in PUTB_μ.

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▶ MARIA BEATRICE BUONAGUIDI, *Strong conditionals for paraconsistent arithmetics: comparisons in proof-theoretic strength.*
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The paraconsistent system of arithmetic developed by Weber [4] using the relevant logic subDLQ seems, on the face of it, to be deductively strong compared to other systems of paraconsistent arithmetic, such as Mortensen's [5]. Weber himself argues that the theory is devised explicitly for classical recapture, but provides no thorough proof-theoretic analysis of its strength. Another system of non-classical arithmetic showing interesting recapture results is HYPE arithmetic HYA, developed by Fischer et al. in [3]. In particular, Fischer et al. show that HYA is proof-theoretically equivalent to PA, and that, using the HYPE conditional, the standard lower bound proofs by Gentzen and Feferman-Schütte for transfinite induction in classical arithmetic and predicative analysis can be reproduced.

In this work, we compare subDLQ arithmetic with HYA proof-theoretically. There are several reasons why this comparison can be significant: indeed, both logics display paraconsistency and have a strong conditional satisfying the Deduction Theorem and Modus Ponens. However, while HYPE is sound and complete with respect to the class of involutive Routley frames [2], subDLQ is only nontrivial and does not have a class of models. We show that, while the strong conditional $\Rightarrow$ of subDLQ allows, similarly to HYPE's $\rightarrow$, to reproduce the Gentzen lower bound proof for transfinite induction in classical arithmetic, the "amount" of paraconsistency we observe in subDLQ does not allow the proof to carry through for all formulae of the full language of arithmetic. We obtain a syntactically definable class Ψ of well-behaved sentences in subDLQ arithmetic for which the proof carries through, but $\Psi \subset \mathtt{Sent}_{\mathcal{L}_{\mathtt{subDLQ}}}$, due to the non-classical behaviour of identity. Conversely, while in HYA, like in subDLQ arithmetic, identity is defined as an equivalence relation, the behaviour of paraconsistency and paracompleteness in HYPE forces identity to behave classically, allowing the proof to go through.

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► GABRIELE BURIOLA, PETER SCHUSTER, ANDREAS WEIERMANN,

Proof-theoretic relations between Higman's and Kruskal's theorem.

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Higman's lemma and Kruskal's theorem are two of the most celebrated results in the theory of well quasi-orders. In his seminal paper [1], G. Higman obtained what is known as Higman's lemma as a corollary of a more general theorem, dubbed here Higman's theorem. J.B. Kruskal was well aware of this more general set up; in the very end of his famous article [2], he explicitly stated how Higman's theorem is a special version, restricted to trees of finite branching degree, of Kruskal's own tree theorem. The equivalence has been subsequently formalized [3]. We transfer Pouzet's proof in the context of Reverse Mathematics, proving its validity over RCA_0 and establishing a rich schema of proof-theoretic implications; moreover, extending the investigations made by Rathjen and Weiermann [4], we calculate the proof-theoretic ordinals of the different versions of Kruskal's theorem involved.

[1] G. Higman, *Ordering by divisibility in abstract algebras*, ***Proceedings of the London Mathematical Society***, vol. 3 (1952), no. 2, pp. 326–336.

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▸ ARTEM BURNISTOV, ALEXEY STUKACHEV, AND MARINA STUKACHEVA, *Computable functionals in Montague semantics.*

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We consider algorithmic properties of mathematical models used in computational linguistics to formalize and represent the semantics of natural language sentences. In particular, finite-order functionals play a crucial role in Montague intensional logic and formal semantics for natural languages [2]. We compare several computable (in sense of [1]) models for the spaces of finite-order functionals based on Ershov-Scott theory of domains and approximation spaces. Namely, we describe how complexity and representability of functional spaces depend from the choice of three basic domains: for entities, for truth values, and for states. This work continues the research started in [3, 4, 5].

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▸ VEDRAN ČAČIĆ, *Tarski's theorem in NFU.*
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The theory NFU is a slight modification of Quine's New Foundations, adjusted in order to have a possibility of Axiom of Choice, and a working proof of consistency relative to ZF. It represents a viable alternative to the "usual" set theory, and it is interesting to see which results of the mainstream theory hold there.

One of these results is Tarski's theorem about equivalence between the squaring principle (any infinite set is equinumerous with its Cartesian square) and the Axiom of choice. Direction ($\Leftarrow$) is proved in roughly the same way as in ZF, however the direction ($\Rightarrow$) presents problems since it uses the principle that for every x there is a $y \notin x$, while obviously $x := V$ (which exists in NFU) cannot have such y. Also, some other details in the proof require a bit more attention. We will explore different ways of proving the missing direction of that theorem.

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▸ WESLEY CALVERT, DOUGLAS CENZER, VALENTINA HARIZANOV, *Generically computable structures.*

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Inspired by the study of generic computability of sets, based on the notion of asymptotic density and introduced in computability theory by C. Jockusch and P. Schupp, we extended such investigation to the context of computable structure theory. We introduced and studied the notion of a generically computable structure and its variants. We say that a countable structure is *generically computable* if it has a substructure the domain of which is a computably enumerable and asymptotically dense set and where the functions and characteristic functions of relations extend to partial computable functions. There are two directions in which this notion of generically computable structures could potentially trivialize: either all structures from a certain algebraic class have generically computable isomorphic copies, or only those having computable (or computably enumerable) copies. While we previously investigated generic and dense computability in general for equivalence structures and for directed graphs induced by one-to-one functions, our more recent focus is on generically computable abelian groups. For example, any (countable) abelian p-group has a generically computable isomorphic copy. We further characterize arbitrary abelian groups that have generically computable isomorphic copies, or other variants of densely computable copies.

▸ DOMENICO CANTONE, PIETRO CAMPOCHIARO, LUCA CUZZIOL, AND EUGENIO G. OMODEO, *Recursively enumerable sets and Diophantine finite-fold-ness*.
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Can we extract, from an available proof that each

$$D_a(x_1, \ldots, x_k) \quad = \quad 0\,, \qquad\qquad a \in \mathbb{N}\,,$$

in some indexed family of equations has at most one solution in $\mathbb{N}$, an effective bound $\mathscr{C}_a$ such that when $x_1 = \boldsymbol{v}_1 \;, \ldots, \; x_k = \boldsymbol{v}_k$ solves $D_a = 0$ then $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_k \leqslant \mathscr{C}_a$? In 1974 (cf. [3]), Yuri V. Matiyasevich provided a negative answer referring to a family of Diophantine equations that involve exponentiation, $u \mapsto 2^u$, and speculated that an alike limitation holds for some collection of polynomials D_a with integral coefficients.

The said limiting result relies, in part, on the fact that the graph

$$\mathcal{F}(a\,,\,b) \quad\Longleftrightarrow\quad F(a) = b$$

of any primitive recursive function $F : \mathbb{N} \longrightarrow \mathbb{N}$ can be specified in the form

$$\exists\, x_1 \cdots \exists\, x_k \quad \varphi(\overbrace{a, b}^{\text{parameters}}\;,\;\overbrace{x_1, \ldots, x_k}^{\text{unknowns}})\,,$$

where φ is an arithmetic formula not involving universal quantifiers, negation, or implication. *Representability* in this form can be achieved even if solely addition and multiplication operators (along with equality and with positive integers) are adopted as primitive symbols of the arithmetic signature; but can representability be reconciled with *univocity*, to wit,

$$\exists\, x_1 \cdots \exists\, x_k\, \forall\, y_1 \cdots \forall\, y_k \left[\varphi(a, b, y_1, \ldots, y_k) \implies \mathop{\&}_{i=1}^{k} (y_i = x_i)\right],$$

or (at worst) with *finite-fold-ness*

$$\exists\, x\, \forall\, y_1 \cdots \forall\, y_k \left[\varphi(a, b, y_1, \ldots, y_k) \implies \textstyle\sum_{i=1}^{k} y_i \leqslant x\right],$$

without calling into play one extra operator designating $u \mapsto 2^u$?

As a preparatory measure towards a hoped-for positive answer to this question, one may consider surrogating the exponentiation operator by a relator $\mathscr{J}(u, v)$ designating an exponential-growth relation (a notion made explicit by Julia Bowman Robinson in 1952). To meet our desiderata, such a relation should be representable in polynomial terms and should link with each u in its domain only a finite number of v's. A promising recipe for constructing such a relation, advanced by Martin Davis in 1968, has been recently reused to construct five new candidate relations. Unfortunately, establishing whether a potential candidate is apt to the job calls for the hard task of proving that at least one of a few special quaternary quartic equations, each corresponding to one of the Heegner numbers $2, 3, 7, 11, 19, 43, 67, 163$, has a finite overall number of integral solutions.

The following synoptic table shows the candidate 'rule-them-all' equations obtained (cf. [1]) through the construction pattern proposed in Davis' 1968 paper [2]. Each such equation is associated with one of the nine so-called Heegner numbers; today we know that proving that any of the quartics

$$
\begin{array}{r||c}
d = 2 & 2\cdot\left(r^2+2\,s^2\right)^2-\left(u^2+2\,v^2\right)^2 \;=\; 1\\
3 & 3\cdot\left(r^2+3\,s^2\right)^2-\left(u^2+3\,v^2\right)^2 \;=\; 2\\
7 & 7\cdot\left(r^2+7\,s^2\right)^2-3^2\cdot\left(u^2+7\,v^2\right)^2 \;=\; -2\\
11 & 11\cdot\left(r^2+r\,s+3\,s^2\right)^2-\left(v^2+v\,u+3\,u^2\right)^2 \;=\; 2\\
19 & 19\cdot 3^2\cdot\left(r^2+r\,s+5\,s^2\right)^2-13^2\cdot\left(v^2+v\,u+5\,u^2\right)^2 \;=\; 2\\
43 & 43\cdot\left(r^2+r\,s+11\,s^2\right)^2-\left(v^2+v\,u+11\,u^2\right)^2 \;=\; 2\;,
\end{array}
$$

associated with the respective Pell equations $x^2 - d\,y^2 = 1$ has only a finite number of solutions in $\mathbb{Z}$ would suffice to ensure that every recursively enumerable set admits a finite-fold polynomial Diophantine representation.

If the equation associated with d is finite-fold, then the following dyadic relation $\mathscr{M}_d$ over $\mathbb{N}$ admits a polynomial Diophantine representation:

$$
\begin{array}{rl}
d\in\{2,7\}: & \mathscr{M}_d(p,q) \;:=\; \exists\,\ell>4\left[\,q=\boldsymbol{y}_{2^\ell}(d)\;\;\&\;\;p\mid q\;\;\&\;\;p\geqslant 2^{\ell+1}\right],\\
d\in\{3,11,19,43\}: & \mathscr{M}_d(p,q) \;:=\; \exists\,\ell>5\left[\,q=\boldsymbol{y}_{2^{2\,\ell+1}}(d)\;\;\&\;\;p\mid q\;\;\&\;\;p\geqslant 2^{2\,\ell+2}\right],
\end{array}
$$

where $\langle \boldsymbol{y}_i(d)\rangle_{i\in\mathbb{N}}$ is the endless, strictly ascending, sequence consisting of all solutions in $\mathbb{N}$ to the said equation $d\,y^2+1=\Box$. Independently of representability, each $\mathscr{M}_d$ turns out to satisfy J. Robinson's exponential growth criteria and Y. Matiyasevich's condition (cf. [5]):

> Integers $\alpha>1$, $\beta\geqslant 0$, $\gamma\geqslant 0$, $\delta>0$ exist such that to each $w\in\mathbb{N}\setminus\{0\}$ there correspond u,v such that: $\mathscr{M}(u\,,\,v)$, $u<\gamma\,w^\beta$, and $v>\delta\,\alpha^w$ hold.

It is very hard to guess whether the number of solutions to any of the six quartics shown above is finite or infinite. For quite a while the authors hoped that Matiyasevich's surmise that each r.e. set admits a single-fold polynomial Diophantine representation could be established by just proving that the sole solution to the quartic $2\cdot\left(r^2+2\,s^2\right)^2-\left(u^2+2\,v^2\right)^2 \;=\; 1$ in $\mathbb{N}$ is $\langle\bar{r},\bar{s},\bar{u},\bar{v}\rangle=\langle 1,0,1,0\rangle$; but Evan O'Dorney (University of Notre Dame) and Bogdan Grechuk (University of Leicester) sent us kind communications that they had found two, respectively three, non-trivial solutions to this equation. The least solution is:

$$
\begin{array}{rcl}
r_1 & = & 877858705853420680629262000814366081842686551436 7,\\
s_1 & = & 1797139324882565197548134105090153037130149943440,\\
u_1 & = & 5221618295817678692343699483662704959631052331713,\\
v_1 & = & 6739958317343073985310999451965479560858521871624;
\end{array}
$$

the components of the third solution are numbers of roughly 180 decimal digits each.

[1] D. Cantone, L. Cuzziol, and E. G. Omodeo, *Six equations in search of a finite-fold-ness proof*, `arXiv:2303.02208` (2023)

[2] Martin Davis, *One equation to rule them all*, ***Transactions of The New York Academy of Sciences. Series II***, vol. 30 (1968), no. 6, pp. 766–773.

[3] Yuri V. Matiyasevich, *Sushchestvovanie neèffektiviziruemykh otsenok v teorii èkponentsial'no diofantovykh uravneniĭ*, ***Zapiski Nauchnykh Seminarov Leningradskogo Otdeleniya Matematicheskogo Instituta im. V. A. Steklova AN SSSR (LOMI)***, vol. 40 (1974), pp. 77–93. (Russian. Translated into English as [4])

[4] ——— *Existence of noneffectivizable estimates in the theory of exponential Diophantine equations*, vol. 8 (1977), no. 3, pp. 299–311. (Translated from [3])

[5] ——— *Towards finite-fold Diophantine representations*, ***Journal of Mathematical Sciences***, vol. 171 (2010), no. 6, pp. 745–752.

▸ JAMES CARR, NICK BEZHANISHVILI, AND TOMMASO MORASCHINI, *Hereditary structural completeness over K4*.

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In deductive systems, a rule is said to be *admissible* iff the tautologies of the system are closed under its applications and *derivable* iff the rule itself holds in the system [3]. Whilst every derivable rule for a system is admissible, whether the converse holds varies between deductive systems. When it does we say the system is *structurally complete*. The classical propositional calculus is structurally complete, whilst many non-classical systems including the intuitionsitic propositional calculus are not [1]. Early invesgitations into structural completeness, indicated it may be possible to precisely characterise the *hereditarily structurally complete* (HSC) systems, SC systems whose finitary extension are also SC. This proved a fruitful question, Citkin [2] produced a characterisation for intermediate logics and Rybakov [4] did so for transitive modal logics. Both these characterisations take a similar form, identifying a small collection of problematic models whose omission is necessary and sufficient for a logic to be HSC.

Recently Bezhanishvili and Moraschini gave a new self-contained proof of Citkin's result by drawing on theory in abstract algebraic logic and Esakia duality [1]. A similar framework exists for transitive modal logics that utilises a duality for modal algebras, and in this talk we explain how one can apply this to obtain a new proof of Rybakov's result. However, more than simply providing this new proof of Rybakov's theorem, the approach illuminates a mistake in Rybakov's characterisation. Accordingly, we establish and prove a revised characterisation of the HSC transitive modal logics.

[1] Bezhanishvili, N. and Moraschini, T, *Hereditarily structurally complete intermediate logics: Citkin's Theorem via Esakia duality*, https://staff.fnwi.uva.nl/n.bezhanishvili/Papers/IPC-HSC.pdf.

[2] Citkin, A., *On structurally complete superintuitionistic logics*, ***Soviet Mathematics Doklady***, vol. 19 (1978) pp. 816-819.

[3] J. Raftery, *Admissible Rules and the Leibniz Hierarchy*, **Notre Dame Journal of Formal Logic**, vol. 57 (2016) pp. 569–606.

[4] Rybakov, V.V., *Hereditarily Structurally Complete Modal Logics*, ***Association for Symbolic Logic***, vol. 60(1995) pp. 266–288.

▸ SZYMON CHLEBOWSKI, *Intuitionistic Logic with Propositional Identity and Propositional Quantifiers.*

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The aim of the talk is to study intuitionistic logic with two additional connectives: propositional identity and propositional quantifiers. The root of our research lies in the work of Roman Suszko devoted to the so called non-Fregean logics [1], which were used to formalize ontology in Wiitgenstein's *Tractatus Logico-Philosophicus*. Suszko introduced propositional identity connective, in the context of classical logic, in order to be able to distinguish between semantic correlates of sentences, which are non longer truth values, but *situations*. In [3] two sequent calculi for minimal non-Fregean logic (SCI system) were presented. Extending these results [2] introduced an intuitive interpretation of propositional identity in the context of intuitionistic logic alongside with Kripke semantics and cut-free sequent calculus for the basic intuitionistic logic with propositional identity (ISCI system). During the talk we will present an extension of the aforementioned system with propositional quantifiers ($\mathsf{ISCI_Q}$). We will discuss intuitive interpretation of the new connectives, Kripke semantics and sequent calculus for the logic $\mathsf{ISCI_Q}$.

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[2] Chlebowski, Szymon *An Investigation into Intuitionistic Logic with Identity*. Bulletin of the Section of Logic, 48(4), 259–283 (2019)

[3] Chlebowski, Szymon *Sequent Calculi for SCI*. Studia Logica, 106,541–563 (2018)

▸ ALAKH DHRUV CHOPRA, FEDOR PAKHOMOV, *Finite labeled trees ordered by non-inf-preserving embeddings.*

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We study the well-quasi-order (wqo) consisting of the set of finite trees with internal and leaf labels coming from arbitrary wqo's P and Q respectively, ordered by homomorphic embeddability which respects the order of the labels. This is a variant of the usual Kruskal ordering but without infima preservation. We calculate the precise maximal order types of these wqo's — in the style of De Jongh, Parikh, and Schmidt — as a function of the maximal order types of P and Q. In doing so, we sharpen some upcoming results of Andreas Weiermann and Harvey Friedman. This also helps to calibrate the reverse mathematical strength of certain well-foundedness assertions and obtain natural combinatorial independence results.

Nash-Williams proved that arbitrary transfinite sequences using finitely many elements from a well-quasi-ordered set are also well-quasi-ordered, but the proof does not offer immediate information about the maximal order type. Erdos and Rado previously proved this for the specific case of sequences of length ω^n using a more concrete approach. Our results lead to precise bounds for transfinite sequences of length less than ω^ω, using the correspondence between the set of labeled finite trees and indecomposable transfinite sequences of finite range with length less than ω^ω. Specific instances of this wqo have also been considered by Marcone and Montalban in their study of a limited form of Fraisse's Conjecture.

[1] D.H.J de Jongh, Rohit Parikh, *Well-partial orderings and hierarchies*, ***Indagationes Mathematicae (Proceedings)***, vol. 80, issue 3, 1977, pp. 195–207.

[2] Diana Schmidt, *Well-Partial Orderings and their Maximal Order Types*, ***Well-Quasi Orders in Computation, Logic, Language and Reasoning***, (Peter M. Schuster, Monika Seisenberger, Andreas Weiermann, editors), Springer Cham., 2020, pp. 351–391.

[3] Harvey M. Friedman, Andreas Weiermann, *Some independence results related to finite trees*, ***Philosophical Transactions of the Royal Society A***, vol. 381, issue 2248.

▸ CIPRIANO JUNIOR CIOFFO, JACOPO EMMENEGGER, FABIO PASQUALI, GIUSEPPE ROSOLINI, *Grothendieck topologies and weak limits for constructive mathematics.*
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The construction of the elementary quotient completion of an elementary doctrine is an excellent tool to produce models of constructive theories for mathematics, see [3, 4]. The construction freely adds quotients of (definable) equivalence relations to an elementary doctrine, which is an algebraic description of a logical theory with equality.

The elementary quotient completion extends the well-known categorical construction of the exact completion $\mathcal{A}_{\mathrm{ex/wlex}}$ of a given category $\mathcal{A}$ with weak limits [1] provided that products are strong. In fact, most of the works that characterise models as exact completions invoke that the given category $\mathcal{A}$ has strong finite products. This matches with the situation of an elementary doctrine, whose base category is required to have finite products. Indeed, equality involves considering pairs of elements.

On the other hand, the peculiarity of strong finite products with respect to weak limits is certainly apparent. In the work [2] for his PhD thesis, one of the collaborators determined suitable conditions to present an extension of the notion of elementary doctrine with respect to a base category $\mathcal{B}$ with just weak finite products. It requires that equality behaves with some kind of bias with respect to a specific weak product diagram—hence the name ***biased elementary doctrine***. He also showed how the elementary quotient completion extends to the wider settings as a 2-functorial left adjoint.

We show how the two extensions refer to the same situation which involves the product completion $\mathcal{A}_{\mathrm{pr}} := (\mathrm{Fam}_{\mathrm{fin}}\,(\mathcal{A}^{\mathrm{op}}))^{\mathrm{op}}$ of a category $\mathcal{A}$. When $\mathcal{A}$ has weak finite limits there is a Grothendieck topology Θ where covers contain a diagram of weak binary products. This observation allows us to state our main results.

THEOREM 1. *Let $\mathcal{A}$ be a category with weak limits. Let P be a doctrine on $\mathcal{A}_{\mathrm{pr}}$ which is a Θ-sheaf. The following are equivalent:*
(i) *The doctrine P is elementary.*
(ii) *The restriction of P to $\mathcal{A}$ is biased.*

THEOREM 2. *Let $\mathcal{A}$ be a category with weak limits. There is a full embedding of categories $\mathcal{A}_{\mathrm{ex/wlex}} \hookrightarrow \mathrm{sh}(\mathcal{A}_{\mathrm{pr}}, \Theta)$ of the exact completion into the topos of Θ-sheaves. The embedding is exact and preserves any local exponential which exists in $\mathcal{A}_{\mathrm{ex/wlex}}$.*

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[3] M.E. MAIETTI AND G. ROSOLINI, *Elementary quotient completion*, ***Theory and Applications of Categories***, vol. 27 (2013), pp. 445–463.

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▶ LUDOVICA CONTI, *Arbitrary Semantics for Abstraction.*
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In this talk, I aim at providing a semantics inspired to Carnap's proposal of an "indirect and partial" interpretation of the theoretical terms of science, in order to model a non-standard interpretation of abstractionist vocabulary, such as the (epistemically) arbitrary one (cf. Magidor 2012). Preliminarly, I present the arbitrary interpretation of the abstract expressions as a clarification of the indeterminacy that they exhibit as *definienda* of the abstraction principles (AP: $\forall F \forall G(@F = @G \leftrightarrow Eq(F, G))$), traditionally considered as implicit definitions. I briefly discuss the advantages of such an interpretation, particularly focusing on the achievements that it provides in terms of logicality of abstraction (cf. Boccuni Woods 2020).

In the first part of the talk, I present a choice-functional reconstruction of the abstractionist theories (AT), following Carnap's strategy. AT is formulated in a second-order abstraction language $L(T^p, T^a)$, where L is second-order logical language, T^p is countable set of first-order and second-order *primitive* constant and T^a is a countable set of *abstract* terms (@F, for any second-order constant) implicitly defined by AP. The first step consists in the elimination of the theoretical terms in $L(T^p, T^a)$ by means of the ramsification of the theory. Considering the language proposition (A) expressing the theory, we obtain, as Ramsification of A, the proposition RS(A): $\exists_1 ... \exists x_n A(T_1^p, ... T_n^p, x_1, ..., x_n)$. The second step consists in the reintroduction of the abstractionist vocabulary through its explicit definition in L_ϵ, namely the language $L(T^p)$ supplemented by the logical ϵ-operator. A complete transposition of AT in a choice-functional theory will be presented.

In the rest of the talk, a model for this theory will be described. It is constituted by an ordered pair $M =< D, I >$, where D is the domain and I is the interpretation function for non-logical vocabulary. Given this background, ϵ is interpreted relative to a given choice function δ for ϵ-terms ($\delta : P(D) \rightarrow D$ s.t. $\forall X \subseteq D\ \delta(X) = x \in X$, if $X \neq \emptyset$; $\delta(X) = x \in D$, if $X = \emptyset$). Then, closed ϵ-terms are interpreted relative to the model M, an assignment function s to variables and the choice function δ on M: $val^{M,\delta,s}(\epsilon x.A(x)) = \delta(val^{M,s}(A(x)) = \delta(\ \{\ d \in D | M, s(x/d) \models A(x)\ \})$ – where A is the set of elements that satisfy AP.

[1] Andreas, Holger, and Georg Schiemer, *A choice-semantical approach to theoretical truth*, ***Studies in History and Philosophy of Science*** Part A 58 (2016), 1-8.

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▸ GRETA CORAGLIA, SHREYA ARYA, ANA LUIZA DA CONCEIÇÃO TENORIO, PAIGE NORTH, SEAN O'CONNOR, AND HANS RIESS, *Categorical methods for fuzzy type theory.*

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We consider classical models for dependent types à la Martin-Löf, such as display map categories [1, 2], and we revisit them in the context of enriched category theory to obtain a fuzzy version of traditional deductive systems. The long-distance scope of this project is to recover many results in opinion dynamics, precisely in the context of cellular sheaves [3], in a fuzzy environment.

Our aim is to – very roughly – model opinions, so we first start by saying what it is that we consider to be an opinion. On this, we follow the path suggested by the correspondence that is mostly known as Curry-Howard: since there is a one-to-one correspondence between logics and programming languages, which kind of looks like the following

proofs	executions (terms)
formula	program (type)

what we do is simply add a leg to it:

proofs	executions (terms)	motivations
formula	program (type)	belief

In our setting, programs are beliefs and executions are thoughts that lead to holding such beliefs. This is of course reductive of the human mind. Nevertheless, we might be able to gain some insight out of it.

Now, given a belief and a motivation for it, someone might consider it good or bad. For example, they could motivate the belief "bees should be protected" either with "they carry pollen between plants" or "I like honey", but perhaps these are not influential to the same extent. This is where fuzziness comes into play.

We present here the first step in the process: we introduce a theory of fuzzy types with their structural rules, and prove soundness and completeness for their calculus. We begin the analysis of possible extension to connectives and type constructors.

This work is part of the Adjoint School project.

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▸ MATTEO CRISTANI, SREEHARI KALOORMANA, AND LUCA PASETTO,
Reasoning about ensemble learning algorithms with Justification Logic.
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An ensemble learning algorithm is a predictive method that uses multiple learning algorithms to obtain better results than it could obtain by using only one of those algorithms[1]. An algorithm, or an ensemble of them, is said to be opaque when its internal workings are not transparent, making it challenging to understand how it makes decisions or to identify the factors that influence those decisions[2].

Justification Logic (specifically LP, the Logic of Proofs) was introduced by Artemov[3]. It allows one to introduce the notion of proofs or justifications in the object language. Instead of writing $\Box X$ to mean that "X is knowable" or that "X is provable", one writes $t : X$ to mean that "t is a justification of X" or that "t is a proof of X"[4].

In this work, we present a first appraisal at reasoning on the opacity of ensemble learning algorithms through Justification Logic, so that a logical explanation of such algorithms can be given.

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[4] Artemov, S. and M. Fitting, ***Justification Logic: Reasoning with Reasons.***, Cambridge Tracts in Mathematics, Cambridge University Press, (2019).

▸ LAURA CROSILLA AND ØYSTEIN LINNEBO, *Definiteness in early set theory*.
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Definiteness played a crucial role in early set theory. In this talk, we bring to light important aspects of definiteness, by distinguishing an intensional and extensional notions of definiteness. We explore different ways of developing these notions by Georg Cantor, Ernest Zermelo and Hermann Weyl. We consider especially Hermann Weyl's proposal. Weyl's interest on definiteness goes back to his "Über die Definitionen der mathematischen Grundbegriffe" [1], where he offered a clarification of the notion of definite property that Zermelo had employed in his axiomatization of set theory in 1908. Weyl's predicative reconstruction of analysis in *Das Kontinuum* [2] made crucial use of a specific rendering of extensional definiteness. We will see how Weyl proposed to construct in a purely logical way, by repeated application of a few principles of construction, extensionally definite properties of the natural numbers. He then took the extensions of those definite properties as key components of the foundation of his predicative analysis.

[1] H. Weyl. Über die Definitionen der mathematischen Grundbegriffe. *Mathematisch-naturwissenschaftliche Blätter*, 7: 93–95 and 109–113.

[2] H. Weyl. *Das Kontinuum: Kritische Untersuchungen über die Grundlagen der Analysis*. Veit, Leipzig, 1918.

▸ VINCENZO CRUPI, TIZIANO DALMONTE, ANDREA IACONA,

Non-monotonicity and Contraposition.

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It is widely acknowledged that an adequate formal theory of defeasible reasoning must not include *Monotonicity*, the principle according to which, if α entails β, then $\alpha \wedge \gamma$ entails β, for any γ. A consequence relation that does not have this property — and so qualifies as non-monotonic — crucially differs from the classical relation of logical consequence. What remains to be settled, however, is which of the other properties of the latter relation should be retained.

In a seminal paper, Gabbay (1985) suggested a restricted set of fundamental properties of non-monotonic logic. His proposal has then been elaborated and refined in different ways. Notably, Kraus, Lehmann, and Magidor (1990) identified a set of properties of non-monotonic systems — known as KLM logic — which included Gabbay's properties. The current literature on non-monotonic logic contains a wide variety of formal theories that develop similar ideas.

There is one point, however, on which most of these theories tend to agree, and which we do not find fully satisfactory: *Contraposition* — the principle according to which, if α entails β, then $\neg\beta$ entails $\neg\alpha$ — is hardly regarded as an essential trait of defeasible reasoning. As far as we can see, no compelling reason has ever been provided for thinking that defeasible reasoning is non-contrapositive.

The line of thought articulated in our paper, accordingly, hinges on the idea that Contraposition is an essential feature of defeasible reasoning. First we define a minimal logic where Contraposition features as the characteristic principle. This logic will be called **E** — for 'evidential' — in line with the terminology adopted by Crupi and Iacona (2022). Then we show some interesting relations that hold in **E** between other principles that have been widely discussed in the literature on non-monotonic logic. Finally, we discuss different ways of strengthening **E**, and provide suitable semantics for each of them.

▸ FRANCESCO DAGNINO, *Quotients in Relational Doctrines.*
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Among categorical approaches to logic, Lawvere's doctrines [2, 3] stand out as a simple and powerful framework capable to cope with a large variety of situations. Doctrines are functors $P : \mathcal{C}^{\mathrm{op}} \to \mathcal{P}os$, on a category $\mathcal{C}$ with finite products, providing an algebraic description of theories in predicate logic: objects and arrows of $\mathcal{C}$ model contexts and terms, products of $\mathcal{C}$ model context concatenation and P maps each object to a poset that models formulas on that object, ordered by logical entailment. Doctrines have proved to be very effective in studying quotients as well as universal constructions freely adding them to any doctrine [4, 5], so that one can use quotients even though they are not natively available.

A longstanding variable-free alternative to predicate logic is the *calculus of relations* [1, 6, 7]. Here one takes as primitive concepts (binary) relations instead of (unary) predicates, with some basic operations, such as relational identities, composition and converse. However natural, this calculus has been much less studied using functorial tools. In this talk we introduce *relational doctrines* as a functorial description of the calculus of relations. We show that relational doctrines are a natural setting where to deal with quotients. Then, we formulate a universal construction that freely adds quotients to any relational doctrine, generalising the elementary quotient completion of an existential elementry doctrine [4, 5]. Moreover, thanks to the variable-free nature of the calculus, we can get rid of products in the base category, thus recovering many new instances, such as, the exact completion of a category with weak finite limits and categories of metric structures, like metric spaces and non-expansive maps.

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▸ MARCELLO D'AGOSTINO, AND COSTANZA LARESE, *Hintikka on the informativeness of logical deduction. A depth-bounded approach to classical first-order logic.*

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In the Sixties and the Seventies [3], Jaakko Hintikka attacks the thesis that logic is analytical and tautological. On the one hand, Hintikka argues that a class of polyadic first-order inferences are synthetic in the sense that it is necessary to introduce new individuals into the argument to derive their conclusion from the premises. On the other hand, Hintikka devises a new notion of information, which he calls *surface information*, that might be increased by logical deduction. In this talk, we individuate some conceptual and technical difficulties affecting Hintikka's work, and propose a different approach, called *Depth-Bounded First-Order Logics* (DBFOLs) [2]. The latter, which extends the propositional natural deduction system of Depth-Bounded Boolean Logics [1], is structured as an infinite hierarchy of logics representing increasing levels of syntheticity or informativeness of classical first-order logic. In particular, we claim that DBFOLs provides: i) a measure of the maximal number of individuals that must be considered together in a certain formula, which improves upon Hintikka's notion of *degree of a formula*; ii) a natural characterization of the intuitive *surface* meaning of the quantifiers, namely, of what it means for an agent to *actually possessing* the information that a quantified formula is true or false; iii) a clear distinction between analytic and synthetic deduction rules, where only the latter introduce new individuals into the argument.

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[2] M. D'Agostino, C. Larese, and S. Modgil, *Towards Depth Bounded First-Order Logics*, ***IfCoLoG Journal of Logics and their Applications***, vol. 8 (2), pp. 423-451.

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▸ RAFAEL DA SILVA DA SILVEIRA, *Precursors of the mathematization of thought applied to logic.*
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By mathematization of thought, understand the slow process that leads to effective formalization in logic, with the aim of employing mathematical linguistic and notational structures, *e.g.* the use of variables, the analysis of combinations and the use of axioms to express an inference; operations with terms that, later, would be linked to logic would become his main means of expression.

Thus, with the objective of understanding how this process developed, the construction of such reflections was carried out, which will be presented in a linear way in relation to the history of philosophy, however, such formalization of logic and thought in the mathematical aspect occurred discontinuously and disperse. For this linearization, aspects related to logic research, formalization and the relationship between mathematics and thought were considered.

Among the thinkers who worked on this project of mathematization of thought and logic, Raimundo Lúlio (1232–1316) and his seven figures of reasoning in *Ars Magna*, Sebastián Izquierdo (1601–1681) and his combinatorial analysis stood out covered in *Pharus Scientiarum*, Thomas Hobbes (1588–1679) and his concept of addition and subtraction in *De Corpore* and Gottfried Wilhelm Leibniz (1646–1716) with his other approach to combinatorial analysis in *Ars Combinatoria*.

[1] Lúlio, R., ***Ars generalis ultima***, Frankfurt: Minerva, 1970.

[2] Lúlio, R., ***The Art of Contemplation***, San Francisco: Ignatius Press, 2002.

[3] Izquierdo, S., ***Pharus Scientiarum.***, Lietard: Lugduni sumpt. Claudii Bourgeat et Mich, 1659. (vol 1).

[4] Leibniz, G.W., ***Dissertatio de arte combinatoria***, Berlim, 1666.

[5] Leibniz, G.W. ***La logic de Leibniz***, Louis Couturat (ed.), Paris: Félix Alcan, 1901.

► MATTEO DE CEGLIE, *The V-logic multiverse and Bencarraf's challenge.*
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Clarke-Doane (2020) argues that the pluralist stance in the philosophy of mathematics, i.e. the position that any consistent mathematical theory produces a legitimate mathematical universe, can provide an answer to Benacerraf (1973) problem iff we interpret it in terms of safety: our set-theoretic beliefs are reliable iff, for any one of them P, we couldn't have easily had a false belief as to whether P. In other words, if and only if we can be safe that by entertaining that belief we are not easily making a mistake. For example, the belief that "$V = L \wedge \exists 0^{\#}$" cannot be held safely, since we have a proof that it is inconsistent, and we cannot have both the conjunctions. However, he also argues that it's not clear how the pluralist can show that her set-theoretic beliefs are safe. In this paper, I argue there is actually a way for the pluralist to show whether her set-theoretic beliefs are safe. To do so, I propose the following, more precise, safety principle:

Principle 1 (Pluralist Safety). *A set theoretic belief φ is safe if and only if it is possible to find a theory T such that $T + \varphi$ is consistent, and there exists an extension of V that witnesses such theory.*

If we were to entertain a belief that φ, but φ cannot be added consistently to any axiomatisation of set theory, then it would be probable that the belief is false, thus not satisfying the Safety principle. At the same time, even if φ could be added consistently to an axiomatisation of set theory, if we still cannot find an extension of V that witnesses this addition we would have doubts on the safety of our belief.

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[2] Benacerraf, Paul, *Mathematical truth*, ***The Journal of Philosophy*** , vol. 70 (1973), no. 19, pp. 661–679.

▸ ANTONIO DI NOLA, GIACOMO LENZI, AND GAETANO VITALE, *Recent results in the spectrum problem.*

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MV-algebras are generalized Boolean algebras where the lattice operations $\vee, \wedge$ are replaced by possibly non idempotent sum and product $\oplus, \odot$. MV algebras are the algebraic counterpart of many valued logic, like Boolean algebras are the counterparts of classical logic. The prime spectrum of MV-algebras is a subject of many investigations in the recent years. At the time of writing there is no completely satisfactory duality for MV-algebras analogous to Stone duality for Boolean algebras. The notion is related also with the theories of lattices and lattice ordered abelian groups. For instance, studying the prime spectra of MV-algebras is equivalent to studying the lattices of principal ideals of lattice ordered abelian groups. We survey some of the most interesting findings.

[1] Lenzi, Giacomo; Di Nola, Antonio, *The spectrum problem for Abelian ℓ-groups and MV-algebras*, ***Algebra Universalis***, vol. 81 (2020), no. 39, pp.43.

[2] Lenzi, Giacomo; Vitale, Gaetano, *Logical complexity of spectra of Abelian ℓ-groups*, ***Journal Of Applied Logics***, vol. 10 (2023), no. 1.

▸ VITALIY DOLGORUKOV, ELENA POPOVA, *Temporal Epistemic Logic for Agents with Delay in Awareness*.
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We present a multi-agent logic for reasoning about knowledge in time which involves delays in the agent's awareness. Awareness framework divides a common epistemic concept of knowledge into explicit and implicit ones. This division helps to avoid the problem of logical omniscience and proposes new instruments for analysis of resource-bounded agents reasoning.

Formulas of the language $\mathcal{L}_T$ are given by the Backus-Naur form

$$\varphi ::= p \mid n_i \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_i\varphi \mid A_i\varphi \mid X\varphi \mid Y\varphi$$

with $p \in Prop$, $i \in Ag$, $n \in \mathbb{N}$.

Formula "n_i" is read as "agent i's delay in awareness is n steps".

A Temporal Epistemic Model for Agents with Delay in Awareness is a tuple

$$\mathcal{M} = (W, (\sim_i)_{i \in Ag}, \rightsquigarrow, A, V, T),$$

where $W \neq \varnothing$ is a set of possible worlds, $\sim_i \subseteq W \times W$ is an accessibility relation for an agent i, $\rightsquigarrow \subseteq W \times W$ is a temporal accessibility relation for the epistemic evolution of the possible worlds, $A_i : W \to \mathcal{P}(\mathcal{L}_T)$ for an agent i, $V : Prop \to \mathcal{P}(W)$ is an evaluation function for propositional variables, $T : Ag \times W \to \mathbb{N}$. Function T (type) returns a natural number, which measures delay in awareness for every possible combination of agent and possible world.

The truth of a modal formula φ in a pointed model $(\mathcal{M}, w)$ is defined as follows:

- the truth of propositional variables and Boolean connectives are defined in a standard way;
- $M, w \models n_i \iff T(i, w) = n$
- $\mathcal{M}, w \vDash K_i\varphi \iff \forall w'(w \sim_i w' \Rightarrow \mathcal{M}, w' \vDash \varphi)$
- $\mathcal{M}, w \vDash X\varphi \iff \forall w'(w \rightsquigarrow w' \Rightarrow \mathcal{M}, w' \vDash \varphi)$
- $\mathcal{M}, w \vDash Y\varphi \iff \forall w'(w' \rightsquigarrow w \Rightarrow \mathcal{M}, w' \vDash \varphi)$
- $\mathcal{M}, w \vDash A_i\varphi \iff \varphi \in A_i(w)$

To illustrate the formal instruments of the system, we will consider this problem with an example of a modified version of the "Muddy Children" puzzle. We introduce different types of agents: fast-reasoners and slow-reasoners, and model different interactions between them given their knowledge. This extension allows to enrich the puzzle's scenarios and bring to the problem of imperfect agency.

In the work, the axiomatization of temporal epistemic logic for agents with delay in awareness is constructed, and soundness and completeness theorem is proven.

▸ FRANCESCA DONEDA, GIUSEPPE PRIMIERO, FRANCESCO A. GENCO, *A many-valued proof-theoretical system for assessing the trustworthiness of information sources.*

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We introduce a graded proof-theoretical system for evaluating trustworthiness among information sources acting under uncertainty. In contexts such as expert debates, with uncertain and constantly updated information, the ground truth is often yet unavailable or undetermined, which makes it impossible to check claims. From these considerations emerges the need to introduce models capable of defining trustworthiness measure on information sources as a useful proxy for establishing their contents' truthfulness. This may represent a helpful tool for laypeople to navigate experts' debates.

We build on a first version presented in [2] which uses the proof theory and relational semantics from [1] to model an information exchange system (a platform where agents can read and write messages) designed for trustworthiness ranking generation. The system relies on a semantic interpretation of positive and negative trustworthiness assessment of messages and several parameters, including fact-checking. To capture the dimension of uncertainty, fundamental in the context of any debate, we now extend this model with a proof theory based on a many-valued logic. This extension significantly increases the expressivity of the model as it allows to evaluate the trustworthiness of a source taking into account the probability to read certain information and its influence on the dynamics involved in its distribution and in the decision to trust it or not. We also offer some basic correspondence results with non-probabilistic calculi.

Keywords. Trust, probabilistic computation, trustworthiness, uncertainty, many-valued logic.

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▸ PABLO DOPICO, *A rose by any other name: more supervalution-style truth without supervaluations*.

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One of the main shortcomings of Saul Kripke's fixed-point semantics based on the Strong Kleene logic, presented for the first time in [1], is that it leaves many logical truths out of the extension of the truth predicate. Thus, as an alternative, Kripke suggests to construct the fixed-point models on the basis of the supervaluationist semantics advanced by van Fraassen [2]. What obtains is a supervaluatonist fixed-point semantics that has arguably constituted one of the most popular solutions to the paradoxes of self-reference.

However, the supervaluationist fixed-point theory of truth is not free from objections. The final picture yields a non-compositional theory of truth whose evaluation scheme is highly intransparent and that cannot be $\mathbb{N}$-categorically axiomatized [3] like the Kripke-Strong Kleene can. For this reason, Johannes Stern [4] has recently advanced a theory (labelled **SSK**) that also meets the goal of the Kripkean supervaluationist theory (i.e., to include all first-order logical truths) while allegedly accounting for the failure of compositionality, and allowing for a $\mathbb{N}$-categorical axiomatization.

Our main contribution in this paper is to show that **SSK** is strikingly similar to a rather understudied theory: Vann McGee's theory of definite truth as presented in [5]. In the first part, we present both theories and prove that McGee's original theory coincides with the minimal fixed point of Stern's theory, modulo a suitable restriction of the language of the former. In the second part, we show how to generalise McGee's theory to any starting inductive set, and prove once again that these theories *à la* McGee coincide with Stern's **SSK** fixed points. In sum, we could say that McGee's theory is an alternative way of obtaining supervaluation-style truth without supervaluations.

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[2] BAS C. VAN FRAASSEN, *Singular terms, truth-value gaps, and free logic.*, ***The Journal of Philosophy***, vol. 63 (1966), no. 17, pp. 481-495.

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► LAURENT DUBOIS, *LDL trans-Referential Theory And Modelization Of Russell Paradox*.

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Initially interpreted as a monster-killer of naive logical theories, Russell paradox can now be seen as an incredible chance in the evolution of science as it led to alternative theories with a strategy of avoidance of paradoxical objects as fruitful as ZFC, VBG, Type Theory, Intuitionism and theories with a strategy of integration like Paraconsistency, Dialetheism.

We suggest an alternative to these alternatives to the naive set and truth theories (non hierarchical T-schema) and develop a first-order theory with the same power as paraconsistent logics, so with a strategy of integration of some paradoxical objects, introducing non-standard objects in well-known mathematical and physical theories.

The LDL Trans-Referential Theory is a kind of dual of the dialetheist theory. The LDL Trans-Referential Project is the opposite of the logicist project : in the LDL Project, the logic is at the service of the mathematical objects.

The fundamental idea of the LDL theory consists in "cloning" universes of standard theories, like, for example, ZF set theory, or number theory, but also of the naive set theory, and in attributing a specific "label" (not a type) to each universe. This is made by way of a cloning function $f : X \to \overset{n}{X}$ and a trans-referential involution function. Then three fundamental axioms (Axiom of Mirroring $\forall \overset{m}{x} \forall \overset{n}{x} (\overset{mm/n}{x} = \overset{n/m}{x} \wedge \overset{nn/m}{x} = \overset{m/n}{x})$ with $m \neq n$) determine the "trans-referential" behavior of the labeled objects, Doing it this way, we can model the Russell paradox.

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▶ THOMAS EHRHARD, FARZAD JAFARRAHMANI, ALEXIS SAURIN, *On denotations of circular and non-wellfounded proofs.*

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This talk investigates the question of denotational invariants of non-wellfounded and circular proofs of the linear logic with least and greatest fixed-points [1]. Indeed, while non-wellfounded and circular proof theory made significant progress in the last twenty years, the corresponding denotational semantics is still underdeveloped.

We will talk about a denotational semantics for non-wellfounded proofs, based on a notion of totality, building on previous work by Ehrhard and Jafarrahmani [2]. Several properties of the semantics will be then discussed: its soundness, the relation between totality and validity and the semantical content of the translation from finitary proofs to circular proofs. Finally, the talk focuses on circular proofs, trying to benefit from their regularity in order to define inductively the interpretation function. It is argued why the usual validity condition is too general for that purpose, while a fragment of circular proofs, strongly valid proofs, constitutes a well-behaved class for such an inductive interpretation.

This talk will be based on a pre-publication available online in which you can find more relevant references.

[1] DAVID BAELDE, AMINA DOUMANE, ALEXIS SAURIN, *Infinitary Proof Theory: the Multiplicative Additive Case*, ***25th EACSL Annual Conference on Computer Science Logic, CSL 2016, August 29 - September 1, 2016, Marseille, France*** (Jean-Marc Talbot and Laurent Regnier, editors), Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2016, pp. 42:1–42:17.

[2] THOMAS EHRHARD, FARZAD JAFARRAHMANI, *Categorical models of Linear Logic with fixed points of formulas*, ***36th Annual ACM/IEEE Symposium on Logic in Computer Science, LICS 2021, Rome, Italy, June 29 - July 2, 2021*** IEEE, 2021, pp. 1–13.

▸ IOANNIS ELEFTHERIADIS, *Algebraically universal categories of relational structures*.

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Since the original theorem of Birkhoff that every group may be realised as the automorphism group of a complete distributive lattice [2], there has been a series of results regarding the representation of groups in various structures. These culminated in the work of Isbell [4], who proposed full embeddings as the means of extending these representation results to a general setting, and lead to the study of *algebraically universal categories*, i.e. those categories that fully embed all categories of universal algebras.

Examples of algebraically universal categories include the categories of graphs, posets, semigroups, distributive lattices, and boolean algebras (all considered with homomorphisms) amongst many others [6]. More recently, Nešetřil and Ossona de Mendez established a partial characterisation of those categories of graphs that are algebraically universal in the framework of finite set theory [5]. This result is based on the combinatorial notion of *nowhere density* and its model theoretic consequences [1].

Motivated by the above, this paper generalises this characterisation to categories of relational structures of arbitrary sizes. This is given in terms of an infinitary variant of nowhere density, and a generalisation of the results in [3]. For the proof, a categorical framework for relational gadget constructions is developed, which is also of independent interest.

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[3] Samuel Braunfeld, Anuj Dawar, Ioannis Eleftheriadis, and Aris Papadopoulos. Monadic NIP in monotone classes of relational structures, 2023.

[4] John Rolfe Isbell. *Subobjects, adequacy, completeness and categories of algebras*. Instytut Matematyczny Polskiej Akademi Nauk, 1964.

[5] Jaroslav Nešetřil and Patrice Ossona de Mendez. Towards a characterization of universal categories. *Journal of Pure and Applied Algebra*, 221(8):1899–1905, 2017.

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▸ DAVID ELLERMAN, *Partition logic and its applications.*
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Propositional logic is a special case of the Boolean logic of subsets (when the universe set is $U = 1$). Subsets and partitions (or quotient sets) are category-theoretic duals so there is a dual logic of partitions. The join and meet operations on partitions were defined in the 19th century (Dedekind and Schr̈oder), but the definition of the implication operation on partitions and two algorithms for defining all the Boolean operations on partition only came in the 21st century [1]. This development of the logic of partitions led to a series of applications.

Just as Boole's first application was the quantitative version of the logic of subsets in finite probability theory, so the first application of partition logic was the quantitative version as the logical information theory based on the notion of logical entropy [2]. The logical entropy of a partition is a probability measure, the probability in two random draws from U that one will draw a distinction of the partition (i.e., a pair of elements in different blocks of the partition). All the usual Venn diagram definitions for simple, joint, conditional, and mutual logical entropy follow from it being a measure. The better-known Shannon entropy is not a measure on a set but there is a non-linear monotonic transformation of those compound logical entropy formulas that yields all the corresponding formulas for Shannon entropy. Thus logical entropy provides a new logical foundation for information theory which includes the Shannon entropy as a specialized formula that is central to coding and communications theory. Logical entropy also generalizes to the quantum level where it is the probability that in two independent measurement of the same quantum state, two different eigenvalues will be obtained.

The second application of partition logic is to the century-old problem of interpreting quantum mechanics (QM). When the set-based concepts of partition mathematics are linearized (a standard method in the mathematics folklore) to vector spaces, e.g., Hilbert spaces, then one arrives at the mathematical formalism of QM [3]–not the physics of QM that must be brought in from classical physics by quantization. Since partitions are the mathematical tool to describe distinctions and indistinctions or distinguishability and indistinguishability, this shows that QM math is the mathematics of (objective) indefiniteness and definiteness. One can even see the lattice of partitions on a set as the bare bones or skeletal version (i.e., stripping away the scalars) of the pure, mixed, and classical states in QM. Moreover, the other basic QM concept is that of an observable operator where the direct-sum decomposition of its eigenspaces is just the linearized version of the inverse-image partition of a numerical attribute $f : U \to \mathbb{R}$ Thus both the quantum states and observables are derived from partitions and the basic notion of measurement, measuring a state by an observable, is represented back at the set level by the join of the two partitions. This way of interpreting the quantum formalism is the "partitional interpretation" of quantum mechanics.

[1] David Ellerman, *The Logic of Partitions: Introduction to the Dual of the Logic of Subsets*, ***Review of Symbolic Logic***, vol.3, no.2, pp.287 - 350.

[2] David Ellerman, ***New Foundations for Information Theory: Logical Entropy and Shannon Entropy***, SpringerNature, 2021.

[3] ——— *Follow the Math!: The Mathematics of Quantum Mechanics as the Mathematics of Set Partitions Linearized to (Hilbert) Vector Spaces*, ***Foundation of Physics***, vol.52, no.5, 100 (2022).

▸ JACOPO EMMENEGGER, FABIO PASQUALI, AND GIUSEPPE ROSOLINI, *Quotients and equality, (co)algebraically, and the elimination of imaginary elements.*
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Doctrines were introduced by Lawvere [2, 3] as an algebraic tool to work with logical theories and their extensions. A doctrine consists of a poset indexed on a category, and the prime example is the Lindenbaum-Tarski algebra of well-formed formulas in context, indexed on the category of contexts (*i.e.* lists of sorts) and substitutions (*i.e.* lists of terms).

The algebraic character of the theory of doctrines makes it a suitable context where to address the question: "What is the theory obtained by (co)freely adding logical structure?" or the closely related question: "How to express additional logical structure in terms of what is already available?". Technically speaking, we look at the 2-category of doctrines and its subcategories on doctrines (read theories) with conjunctions, or with equality, or with all connectives and quantifiers from first order logic, and then ask whether whether a certain "forgetful" functor is adjoint and, in the second case, whether the adjunction obtained in this way is (co)monadic.

After an introduction to doctrines and their connection to logic and type theory, I shall present the main results of [1] discussing the above questions in the case of two forgetful functors: the one from theories with conjunctions, equality and quotients to theories with conjunctions and equality, and the one that further forgets equality. Not surprisingly, the answers revolve around the concept of equivalence relation. I shall discuss applications to useful constructions in categorical logic and type theory, as well as to the elimination of imaginary elements in the sense of Poizat [4]. If time allows, I shall also describe how to lift this setting to Grothendieck fibrations (of which doctrines are a particular case) using groupoids instead of equivalence relations.

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▸ FREDRIK ENGSTRÖM AND ORVAR LORIMER OLSSON, *The propositional logic of teams.*

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Starting from a logic given by traditional semantics formulated in terms of semantic objects (i.e., assignments, valuations or worlds) *team semantics* lifts the denotations of formulas to sets, or *teams*, of semantic objects instead enabling the formulation of properties, such as variable dependency, not available in the traditional setting. Since the introduction by Hodges, and refinement by Väänänen, team semantic constructions have been used to generate expressively enriched logics still conserving nice properties, such as compactness or decidability [1]. In contrast these logics fail to be substitutional, limiting any algebraic treatment, and rendering schematic proof systems impossible. This shortcoming can be attributed to *the flatness principle*, commonly adhered to when generating team semantics [2].

Investigating the formation of team logics from algebraic semantics, and disregarding the flatness-principle, we present *the logic of teams* (LT), a substitutional logic for which important propositional team logics are axiomatisable as fragments. Starting from classical propositional logic and Boolean algebras, we give semantics for LT by considering the algebras of the form $\mathcal{P}B$ for a Boolean algebra B, treated with an *internal* (derived from B) and an *external* (set-theoretic) set of connectives. Furthermore, we present a well-motivated labelled natural deduction system for LT, for which a further analysis motivates a generalisation to constructions of logics by combinations of an internal and an external logic, where for LT both are classical propositional logic.

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[2] ORVAR LORIMER OLSSON, ***Monadic semantics, team logics and substitution***, Master's thesis, University of Gothenburg, 2022, hdl.handle.net/2077/72005.

▶ DAVIDE FALESSI AND FABIEN SCHANG, *Thinking about Being, Existence, and Nothingness: Why 'Every thing' is not 'Everything' (among other 'thing's).*

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The meaning of a special class of statements is studied in the present talk: quantifying statements, i.e. those statements including two occurrences of expressions like 'every', 'some', and 'no'. The talk is divided into two parts. A first general introduction is devoted to being and nothingness. First of all, an explanation of the several senses of "being" and the existential quantifier (quantifier variance) will be provided, showing the difference between a monist and a pluralist account of existence. Secondly, a distinction between the various senses of "nothingness" will be considered, examining its categorematic and syncategorematic uses in relation with "being". A second part will show that second order logic is required to make sense of quantifying statements by using mixed quantifiers that range both on individuals and predicates. Then an exhaustive set of logical relations is gathered by syntactic and semantic means, and a set-theoretical representation is proposed to make a calculus of logical relations in terms of ordered semantic values: this is a special numbering semantics for quantifying statements, where the logical value of a statement corresponds to the set of its models. This will be done by computing the set of logical relations between such formulas into an extended version of the theory or relations. Finally, a number of other kinds of formula will be mentioned that can be also explained into such numberings: categorical propositions, dyadic sentences, Barcan formulas, or epistemic statements. The general semantic method will be sketched with its two main strategies to construct characteristic numberings: language-adaptive, and model-adaptive strategies

▸ FEDERICO FAROLDI, ATEFEH ROHANI, AND THOMAS STUDER, *Conditional obligations in justification logic.*
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Standard deontic logic faces problems in representing conditional and contrary-to-duty obligations, such as those formulized by the well-known Chisholm's puzzle. However, dyadic deontic logic overcome this puzzle by introducing dyadic obligations [6]. Since justification logics [1, 5] have successful background in deontic contexts [3, 2], we present a justification counterpart for dyadic deontic logic. Firstly, the alethic-deontic system E is considered [4], and then an explicit version of this system, JE is introduced by replacing the alethic $\Box$-modality with proof terms and dyadic deontic $\bigcirc$-modality with justification terms [7]. Notably, the explicit representation of strong factual detachment (SFD) is given and finally, soundness and completeness of system JE with respect to basic models and preference models is established.

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▶ DAVID FERNÁNDEZ-DUQUE, AND KONSTANTINOS PAPAFILIPPOU, *On the expressive completeness of a tangle operator over the topological μ-calculus.*
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The modal μ-calculus $\mathcal{L}_\mu$ is a decidable, but very expressive extension of modal logic as it embeds many modal/temporal logics such as PDL, CTL and CTL^*. Dawar and Otto [1] have shown a modal characterisation theorem for $\mathcal{L}_\mu$ over $\mathsf{K4}$ frames. In particular, they proved that a weaker logic $\mathcal{L}_{\diamond^\infty}$ is equivalent to the bisimulation invariant part of MSO. The tangle logic $\mathcal{L}_{\diamond^\infty}$ is defined by replacing the fixed point operator with the tangle derivative operator which is defined on a finite set of formulas Γ as $\diamond^\infty\Gamma := \nu x. \bigwedge_{\varphi\in\Gamma} \diamond(\varphi \wedge x)$. It is of significance in the study of topological modal logic as it corresponds to an extension of the notion of the perfect core of a set. Furthermore, similar questions of the expressivity of tangle operators arise for arbitrary topological derivative spaces which are the topological semantics characterised by the logic of so-called weakly transitive frames $\mathsf{wK4}$.

An analysis of Baltag, Bezhanishvili and Fernández-Duque [2] gave completeness and weak FMP for $\mathcal{L}_\mu$ over $\mathsf{wK4}$ frames as well as topological derivative spaces. This was done by using the notion of finality, where a world w is final in a model M for some formula ϕ if $M, w \vDash \phi$ and given u with wRu and $M, u \vDash \phi$ then uRw. They show that we cannot have the same kind of modal characterisation theorem of $\mathcal{L}_\mu$ over $\mathsf{wK4}$.

At the same time we know that $\mathcal{L}_{\diamond^\infty}$ is not as expressive as $\mathcal{L}_\mu$ over $\mathsf{wK4}$ frames. We use the notion of finality and the finite model property to show that a slightly more expressive tangle operator does in fact provide a logic equivalent to that of $\mathcal{L}_\mu$ over $\mathsf{wK4}$ frames. As a corollary we get a model theoretic proof of the collapse of the alternation hierarchy over $\mathsf{wK4}$ frames, a result which was recently shown using proof theoretic methods in [3].

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► CAMILLO FIORE, JOAQUÍN TORANZO CALDERÓN, *A family of contrastructural classical logics*.
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The standard presentation of multiple-conclusion classical logic stipulates that an argument is valid just in case whenever *all* the premises are true *some* conclusion is true. Thus, it imposes what we call an *existential* reading of conclusions and a *universal* reading of premises. This is the orthodox reading of multiple-conclusion consequence in philosophical logic, but it is not the only one possible. In this talk, we introduce and study a family of logical systems, each of which results by taking multiple-conclusion classical logic and modifying the definition of validity so as to induce a specific (often heterodox) reading of premises and/or conclusions. More precisely, the family consists of the 16 systems that can be defined by means of the following schema:

$$\Gamma \models^{Q_1 Q_2} \Delta \quad \text{iff} \quad \forall v \in \mathcal{V}: \quad \text{if } (Q_1\, \gamma \in \Gamma)\ v(\gamma) = 1 \text{ then } (Q_2\, \delta \in \Delta)\ v(\delta) = 1$$

where Γ and Δ are sets of sentences, $\mathcal{V}$ is the set of classical interpretations of the language, and Q_1 and Q_2 are any quantifiers in the set $\{\forall, \exists, \hat{\forall}, \hat{\exists}\}$, with $\hat{\forall}$ and $\hat{\exists}$ defined thus:

$$\begin{aligned}(\hat{\forall}\ \sigma \in \Sigma)\phi &\quad := \quad \Sigma \neq \varnothing \wedge (\forall \sigma \in \Sigma)\phi \\ (\hat{\exists}\ \sigma \in \Sigma)\phi &\quad := \quad \Sigma = \varnothing \vee (\exists \sigma \in \Sigma)\phi\end{aligned}$$

(From an intuitive standpoint, $\hat{\forall}$ and $\hat{\exists}$ can be understood as restricted quantifiers with and without existential import on their domain of quantification, respectively.) When Q_1 is $\forall$ and Q_2 is $\exists$, the above schema delivers standard multiple-conclusion classical logic, here denoted $\mathbf{CL}^{\forall\exists}$. For all other cases, the resulting systems deviate from $\mathbf{CL}^{\forall\exists}$ both in their valid inferences and in their structural properties; interestingly, they not only lack some structural properties that classical logic enjoys but also enjoy some structural properties that classical logic lacks. This is why we call them *contrastructural classical logics*.

The talk has three main parts. In the first part, we study our logics from a model-theoretic standpoint; we show how they are ordered by inclusion and analyze their structural properties. In the second part, we study our logics from a proof-theoretical standpoint; we provide a recipe for constructing a sound and complete sequent calculus for each of them. Lastly, in the third part, we discuss the informal interpretation and potential applications of the systems presented; we argue that they can be viewed as allowing the application of classical reasoning to different epistemic contexts (where the agent has particular informational resources and goals).

▸ HAO-CHENG FU, *AGM theory and the Ramsey test.*

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In AGM theory, there are two important issues that remain to be resolved. One is the incompatibility between the Ramsey test and the Preservation principle, and the other is the problem of iterated belief change. These two issues are related: the Ramsey test is an attempt to explain the rationality of belief revision function, but with it and the Preservation principle one will derive the triviality in AGM theory. Gärdenfors [6] blames that the culprit is the Ramsey test and contends that we should give up explaining the role of conditional in the theory of belief revision by virtue of Ramsey test. Some scholars such as Bradley [1],[2] and Chandler [3],[4] provide different ideas to this problem. They contend that the Ramsey test should be retained, and raise the challenges to the Preservation principle, especially in the subject of the DP problem. This paper aims to examine the conflict between these two solutions and prove that the triviality problem and the DP problem cannot be solved by weakening the Ramsey test or the Preservation principle.

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▸ FRANCESCO GALLINARO AND JONATHAN KIRBY, *Quasiminimality of complex powers.*

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A conjecture due to Zilber predicts that the complex exponential field is quasiminimal: that is, that all subsets of the complex numbers that are definable in the language of rings expanded by a symbol for the complex exponential function are countable or cocountable. Zilber showed that this conjecture would follow from Schanuel's Conjecture and an existential closedness-type property asserting that certain systems of exponential-polynomial equations can be solved in the complex numbers; later on, Bays and Kirby were able to remove the dependence on Schanuel's Conjecture, shifting all the focus to the existence of solutions. In this talk, I will discuss recent work about the quasiminimality of a reduct of the complex exponential field, that is, the complex numbers expanded by multivalued power functions.

► RENÉ GAZZARI, *Defining Natural Deduction derivations using a formal theory of occurrences.*
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Distinctive feature of Gentzen's [2] Natural Deduction is the possibility of *discharging* previously assumed statements. A well-defined representation of such a facultative discharge (as proposed by Gentzen) requires the notion of *occurrences* of assumptions and inference steps in derivations. Even though Gentzen and his successors, in particular Prawitz [3], were aware of the necessity to deal with occurrences (instead of considering only the syntactic entities themselves), little to nothing more than an intuitive account is found in the literature. Only recently, Gazzari [1] developed a full-fledged formal theory of occurrences capable of representing occurrences in a generality needed for adequately defining the discharge function and, this way, derivations.

We provide relevant concepts of the theory of occurrences and, based on these concepts, central proof-theoretic definitions, as of assumptions, inference steps, discharge functions and, in the end, of derivations. This is a foundational topic, of conceptual character, whose purpose is to give insight into the underlying technical complexity of the usual intuitive treatment of derivations and the assurance that a detailed formal definition of this intuitive treatment can be obtained.

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▶ FRANCESCO A. GENCO, *Probabilistic computation and trust through the lens of typed λ-calculus* (a joint work with GIUSEPPE PRIMIERO).
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Computational systems evolve at a vertiginous speed and our notion of computation evolves with them. In recent years, the theory of computation embraced the study of systems that lie certainly outside its traditional borders. Probabilistic programming languages are a most prominent example of this phenomenon. The broadening of our notion of computation does not only force us to consider new mechanisms and systems, but also to rethink, in some cases, the role of some fundamental and well-established notions. The traditional notion of *correctness*, for instance, does not fare well at all with respect to probabilistic programs. A typical probabilistic program, indeed, cannot be said to compute *the* correct result. In spite of this, we often have quite strong expectations about the frequency with which it should return different outputs. Hence, instead of requiring programs to be *correct*, we can appeal to a weaker notion and require them to be *trustworthy*, where by *trustworthy* we mean that they yield outputs as determined by probability distributions that model their expected behaviour.

We present a computational framework that precisely formalises this intuition. In order to do so, we define a typed λ-calculus that features operators for conducting experiments on probabilistic programs and for evaluating whether the frequency of their outputs is compliant with target probability distributions. We then discuss the fundamental computational properties of the calculus: subject reduction, progress and termination.

The analysis of the notion of trust provided by the calculus is essentially pragmatic in nature and, coherently, heavily depends on the contingencies of the particular program execution under consideration. In order to also provide a more abstract, atemporal characterisation of the investigated phenomena, we introduce a notion of *confidence* which does not depend on the development of any particular execution of a program but only on its static definition. This enables us to prove—by an application of the strong law of large numbers—that the calculus and the notion of trust formalised in it globally behave as expected with respect to the basic tenets of probability theory.

Keywords. Trust, probabilistic computation, type theory, lambda-calculus.

▸ GUIDO GHERARDI, AND EUGENIO ORLANDELLI, *Logics of super-strict implications.*

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C.I. Lewis' [3] strict implication ($\prec$) is a stregthening of material implication ($\supset$) that avoids its paradoxes—$\neg B \supset (B \supset A)$ and $A \supset (B \supset A)$. It is meant to provide a formal explication of entailment-related uses of implication. Connexivists [4] and relevantists [1] have argued that the paradoxes of strict implication—$\bot \prec A$ and $B \prec \top$—are a reason to discard $\prec$ and they have proposed alternative implications that are paradox-free. One limitation of their proposals is that they involve a major departure from classical logic.

Super-strict implication ($\rhd$) strengthens $\prec$ in order to avoids its paradoxes: $A \rhd B$ is true whenever $A \supset B$ is necessary and A is possible, see [2]. In this way we obtain a paradox-free implication that is compatible with classical logic. This talk provides some motivations for $\rhd$ and studies proof-systems for some important logics of $\rhd$.

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▸ DAVID GONZALEZ, *The ω-Vaught's conjecture*.
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Robert Vaught conjectured that the number of countable models of any given list of axioms must be either countable or continuum, but never in between. Despite all the work that has gone into this conjecture over the past sixty years, it remains open. It is one of the most well-known, long-standing open questions in mathematical logic. We introduce the ω-Vaught's conjecture, a strengthening of Vaught's conjecture for infinitary logic. We believe that a structural proof of Vaught's conjecture for infinitary logic would actually be a proof of the ω-Vaught's conjecture. Furthermore, a counterexample to the ω-Vaught's conjecture would likely contain ideas helpful in constructing a counterexample to Vaught's conjecture.

We prove the ω-Vaught's conjecture for linear orderings, a strengthening of Vaught's conjecture for linear orderings originally proved by Steel [Ste78]. The proof notably differs from Steel's proof (and any other previously known proof of Vaught's conjecture for linear orderings) in that it makes no appeal to lemmas from higher computability theory or descriptive set theory.

This talk is based on joint work with Antonio Montalbán.

[Ste78]John R. Steel. On Vaught's conjecture. In *Cabal Seminar 76–77 (Proc. Caltech-UCLA Logic Sem., 1976–77)*, volume 689 of *Lecture Notes in Math.*, pages 193–208. Springer, Berlin, 1978.

▸ VALENTIN GORANKO, *On modal logics with weakly transitive accessibility relations.*
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Say that binary relation R on a set W has the *weak transitivity property* if for every $u, v \in W$, if v is reachable from u in 3 R-steps, then v is reachable from u in 2 R-steps. Formally, weak transitivity is expressed by the first order sentence

$$\forall u \forall v (\exists x \exists y (uRx \wedge xRy \wedge yRv) \rightarrow \exists z (uRz \wedge zRv)).$$

Clearly, every transitive relation is weakly transitive, but not vice versa. Non-transitive but weakly transitive relations do not naturally occur often, but there are several interesting and diverse cases where they do, including:

- The edge relation in the countable random graph (aka, Rado graph); more generally, in any graph of diameter 2.
- The comparability relation between nodes in forests, regarded as models of branching time, where that relation can be naturally defined as "*being on the same history (timeline)*".
- The right (or left) neighbourhood relation between two intervals on a linear order, where an interval j is a right neighbour of the interval i iff the end of i and the beginning of j coincide.

Weak transitivity is frame-definable by the modal formula $\Diamond\Diamond\Diamond p \rightarrow \Diamond\Diamond p$ or, equivalently, by $\Box\Box p \rightarrow \Box\Box\Box p$. Added as an axiom to the modal logic $\mathbf{K}$ it defines the simplest normal modal logic, $\mathbf{K}_3^2$, for which, to my knowledge, no published works yet have proved or disproved decidability, nor finite model property.

This work presents and discusses some modal logics (including those associated with the cases above) containing $\mathbf{K}_3^2$ but not $\mathbf{K}_4$ (i.e., with weakly transitive but generally non-transitive relations), including some known and some new results about representation theorems, finite model property, and decidability for them.

▸ ARIEL GRUNFELD, *Monadic realizability for intuitionistic higher-order logic.*
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The standard construction for realizability semantics of intuitionistic higher-order logic is based on partial combinatory algebras as an abstract computation model with a single computational effect - non-termination. Recent work [1] demonstrated how generalizing the model can affect the validity of formulas in the language, and suggested the general framework of evidenced frames [2] for constructing realizability models, using stateful and non-deterministic computation as examples. As first discussed in [3], many computational effects can be modeled using monads, where programs are interpreted as morphisms in the corresponding Kleisli category. To account for a more general notion of computational effects, we construct an evidenced frame where the underlying computational model is defined in terms of an arbitrary monad, generalizing partial combinatory algebras to combinatory algebras over a monad, and using monotonic post-modules to relate predicates to computations.

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▸ PIOTR GRUZA, *The structure of the definability relation between definitions of truth*.
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A theory T is a *theory of truth* for a language $\mathcal{L}$ if and only if there exists a formula Θ such that for each sentence $\sigma \in \mathcal{L}$, the theory T proves $\Theta(\ulcorner\sigma\urcorner) \leftrightarrow \sigma$. The well-known theorem of Alfred Tarski states that a reasonably strong theory cannot be a theory of truth for its language.

In this talk, we focus on finitely axiomatizable theories of truth (called *definitions of truth*) for the language of Peano Arithmetic extending $\mathrm{I}\Delta_0 + \mathrm{Exp}$. For simplicity, we assume that all considered theories are expressed in languages extending $\mathcal{L}_{\mathrm{PA}}$ by relational symbols only.

Having two theories of truth S and T, we say that S *defines* T iff we can assign to every non-arithmetic n-ary symbol $\mathfrak{R}$ of $\mathcal{L}_T$ an n-ary formula $\Theta_{\mathfrak{R}}$ of $\mathcal{L}_S$ in such a way that S proves every axiom of T with $\Theta_{\mathfrak{R}}$ substituted for each occurrence of $\mathfrak{R}$ for each symbol $\mathfrak{R}$ – in other words, S directly and conservatively over $\mathcal{L}_{\mathrm{PA}}$ interprets T. It can be seen that a definability relation constitutes a preorder on the theories of truth ($S \geq T$ iff S defines T).

Using a method developed by Fedor Pakhomov and himself, Albert Visser showed that there is no minimal element in the definability preorder among definitions of truth. Combining that method with some truth-theoretic techniques, we prove that the order generated by that preorder is a distributive lattice which embeds every countable distributive lattice.

Joint work with Mateusz Łełyk.

▸ STEFAN HETZL, JANNIK VIERLING, *Proof-theoretic analysis of automated inductive theorem proving.*

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Automating the search for proofs by induction is an imporant topic in computer science with a history that stretches back decades. A variety of different approaches and systems has been developed. Typically, these systems have been evaluated empirically and thus very little is known about their theoretical limitations.

In this talk I will present a proof-theoretic approach for understanding the power and limits of methods for automated inductive theorem proving. A central tool are translations of proof systems that are intended for automated proof search into (very) weak arithmetical theories. This allows not only to locate a method in a partial order of theories but also to provide examples for unprovable statements which are of practical interest in computer science.

This research gives rise to a number of new problems and questions about (very) weak aritmetical theories, mostly concerning unprovability results.

[1] Stefan Hetzl and Jannik Vierling, *Induction and Skolemization in saturation theorem proving*, ***Annals of Pure and Applied Logic***, 174(1):103167, 2023.

[2] Stefan Hetzl and Jannik Vierling, *Unprovability results for clause set cycles*, ***Theoretical Computer Science***, 935, pp. 21-46, 2022

[3] Jannik Vierling, ***The limits of automated inductive theorem provers***, PhD thesis, TU Wien, Austria.

▸ ÅSA HIRVONEN AND JONI PULJUJÄRVI, *Finite Ehrenfeucht–Fraïssé games of continuous logic*.
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Continuous first-order logic is a natural logical framework, defined in [1], for the logical treatment of metric structures, i.e. structures based on a bounded complete metric space rather than a pure set.

We define a version of finite length Ehrenfeucht–Fraïssé games for continuous first-order logic and show that the second player having a winning strategy in a game of length n between two structures corresponds exactly to the two structures being equivalent with respect to sentences of quantifier rank $\leq n$, as is also the case in classical first-order logic. Our game is inspired by our prior work on dynamic and infinite EF games for specific classes of metric structures [2]. Some prior study on EF games for continuous logic has been done e.g. in [3], but there does not seem to be a connection to quantifier rank.

We then proceed to show some ways of using the newly defined games to prove that certain properties of structures are not definable by a sentence of continuous first-order logic. Due to the real-valued nature of the logic, using the game for this purpose is not completely analogous to the classical case.

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[2] Åsa Hirvonen and Joni Puljujärvi, *Games and Scott sentences for positive distances between metric structures*, ***Annals of Pure and Applied Logic***, vol. 173 (2022), no. 7.

[3] Bradd Hart, *Ehrenfeucht-Fraïssé games in continuous logic*, presentation notes (2018), https://ms.mcmaster.ca/ bradd/EF-games.pdf

▸ ANDREA IACONA AND PAOLO MAFFEZIOLI, *Intuitionistic evidential conditional.*
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In some recent works, Crupi and Iacona have developed an account of conditionals — the *evidential account* — which rests on the idea that a conditional is true when its antecedent is incompatible with the negation of its consequent. This incompatibility condition, which is intended to capture the intuition that the antecedent of a conditional must support its consequent, is spelled out in terms of a classical modal semantics that allows for comparative measures of distance between worlds.

Although the choice of a classical framework has some obvious advantages, and makes the results so obtained directly comparable with standard theories of conditionals such as those provided by Stalnaker and Lewis, it is arguable that the evidential account could equally be framed in a different framework. In particular, the idea that a conditional is true when its antecedent is incompatible with the negation of its consequent is to a large extent neutral as to the distinction between classical and intuitionistic logic.

The aim of our paper is to show precisely how the evidential account can be developed within an intuitionistic framework. We start by extending the language of propositional intuitionistic logic with a new connective $\rhd$ for evidential conditional. Then, we define a Kripke model M for such a language as an ordered tuple $\langle W, A, \prec, V\rangle$ where W is a nonempty set, A assigns to each $x \in W$ a subset W_x of W such that (i) $x \in W_x$ and (ii) if $y \in W_x$ and $z \in W_y$, then $z \in W_x$, $\prec$ assigns to each $x \in W$ an irreflexive and transitive relation $\prec_x$ on W_x, and finally V is the usual valuation function satisfying the intuitionistic heredity condition. Moreover, let $Min_x(S)$ be the set of all $y \in S \cap W_x$ for which there is no $z \in S \cap W_x$ such that $z \prec_x y$. In these models, an evidential conditional $\alpha \rhd \beta$ is evaluated as follows:

$[\alpha \rhd \beta]_x = 1$ iff for every $y \in W_x$, if $[\alpha]_y = 1$ and $[\beta]_y = 0$, then

(a) some $z \in Min_x$ is such that $[\alpha]_z = [\beta]_z$;
(b) for every $z \in Min_x(\alpha)$, $[\beta]_z = 1$;
(c) for every $z \in Min_x(\neg\beta)$, $[\neg\alpha]_z = 1$.

whre $Min_x(\alpha)$ and Min_x are abbreviations for $Min_x(||\alpha||)$ and $Min(W_x)$, respectively.

We will show that, once Kripke models are appropriately constrained, we get an adequate semantics for $\rhd$. Firstly, we provide a complete map of the deductive relationships between the intuitionistic, classical and evidential conditional. Secondly, we compare the intuitionistic conditional *vis-a-vis* the evidential conditional with respect some notable properties discussed in the literature on conditionals such 'contraposition', 'true consequent', 'conditional proof', 'conditional excluded middle', 'conjunction sufficiency', etc. From all this we conclude that on the backgrouond of intuitionsitic logic, the evidential conditional is an intersting and novel generalization of the intuitionstic conditional.

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▸ ASSYLBEK ISSAKHOV, ULDANA OSTEMIROVA, *Notes on hyperimmunity and computably enumerable equivalence relations.*
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Keywords: computably enumerable equivalence relation, hyperimmune set, universal ceer.

If R, S are equivalence relations on the set ω of the natural numbers, we say that R is computably reducible to S (notation: $R \leq S$) if there exists a computable total function f such that, $(\forall x, y \in \omega)(xRy \Leftrightarrow f(x)Sf(y))$, [1].

For every set A, let $xR_Ay \Leftrightarrow (x = y or x; y \subseteq A)$, and let $xIdy \Leftrightarrow x = y$. An infinite set A is hyperimmune if and only if no computable function f majorizes A. A function f majorizes an infinite set A if f majorizes its principal function p_A (i.e. $f(x) \geq p_{A(x)}$ for all $x \in \omega$), where $p_{A(n)} = a_n$ for $A = \{a_0 < a_1 < a_2 < \cdots\}$. It is known that $deg(Id)$ consists of all decidable computably enumerable equivalence relations (further - ceers) with infinitely many equivalence classes, and if $Id \leq R \leq R_A$ then there exists a c.e. set B such that $R \equiv R_B$, [2]. Some interesting recent properties of hyperimmunity and numberings one can find in [3].

LEMMA 1. *If A is a hyperimmune set, then $Id \nleq R_{\bar{(}A)}$.*

For a given partial computable function φ, let P_φ be the ceer defined in the following way: $xP_\varphi y \Leftrightarrow (x = y or \varphi(x) \downarrow = \varphi(y) \downarrow)$.

Let's denote by PC the class of all ceers of the form P_φ. Let
$xH_1y \Leftrightarrow (x = y or \varphi_x(y) \downarrow = \varphi_y(x) \downarrow)$.

A ceer R is universal if $S \leq R$ for any ceer S. It is known that universal ceers do exist, [4].

THEOREM 2. *If a relation $R \in PC$, then $R \leq H_1$.*

[1] YU. L. ERSHOV, *Positive equivalences*, ***Algebra and Logic***, vol. 10 (1973), no. 6, pp. 378–394.

[2] U. ANDREWS AND A. SORBI, *Joins and meets in the structure of ceers*, ***Computability-The journal of the association CIE***, vol. 8 (2019), no. 3-4, pp. 193–241.

[3] F. RAKYMZHANKYZY, N.A. BAZHENOV, A.A. ISSAKHOV, B.S. KALMURZAYEV, *Minimal generalized computable numberings and families of positive preorders*, ***Algebra and Logic***, vol. 61 (2022), no. 3, pp. 188–206.

[4] U. ANDREWS, S. LEMPP, J. S. MILLER, K. M. NG, L. SAN MAURO, AND A. SORBI, *Universal computably enumerable equivalence relations*, ***Journal of Symbolic Logic***, vol. 79 (2014), no. 1, pp. 60–88.

▶ ALEKSANDER IVANOV, MONIKA DRZEWIECKA AND BARTOSZ MOKRY,
Generics in invariant subsets of automorphisms of homogeneous structures.
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Let M be a countable ultrahomogeneous structure and let $\rho \in \mathsf{Aut}(M)$. Let $\mathcal{C}_\rho = cl(\rho^{\mathsf{Aut}(M)})$, where $\rho^{\mathsf{Aut}(M)}$ denotes the conjugacy class of ρ and cl denotes the operation of topological closure in the automorphism group. We study the following question: *When does the subspace $\mathcal{C}_\rho$ contain a conjugacy class of $\mathsf{Aut}(M)$ which is comeagre in it?* Having an answer to this question for all $\rho \in \mathsf{Aut}(M)$ we, in fact, describe all closed subsets $\mathcal{C} \subseteq \mathsf{Aut}(M)$ which are invariant under conjugacy in $\mathsf{Aut}(M)$ and have comeagre conjugacy classes.

Let $\mathcal{P}$ be the set of all finite partial isomorphisms of M. The set $\mathcal{P}$ is ordered by the relation of extension of maps. In this terms we can formulate the standard definitions of the joint embedding property (JEP), the amalgamation property (AP), the cofinal amalgamation property (CAP) and the weak amalgamation property (WAP), see [2]. Let $\mathcal{P}_\rho = \{p \in \mathcal{P} : p \text{ extends to an automorphism from } \mathcal{C}_\rho\}$. We deduce from [1]:

> The set $\mathcal{C}_\rho$ has a comeagre conjugacy class if and only if the family $\mathcal{P}_\rho$ has WAP.

Since for all known examples of structures M and automorphisms $\rho \in \mathsf{Aut}(M)$ with comeagre conjugacy classes in $\mathcal{C}_\rho$, the family $\mathcal{P}_\rho$ has CAP, we also ask: *Is it true that in this context properties WAP and CAP are equivalent?*

Let G be a closed highly homogeneous subgroup of S_∞ not involving circular orderings. We show that any $\mathcal{C}_\rho$ from G contains a conjugacy class which is comeagre in it. Furthermore, the corresponding $\mathcal{P}_\rho$ has the cofinal amalgamation property. In the case of the automorphism group of a typical ultrahomogeneous partially ordered set similar results are proved.

[1] A. Ivanov, *Generic expansions of ω-categorical structures and semantics of generalized quantifiers*, ***The Journal of Symbolic Logic***, vol. 64 (1999), pp. 775–789.

[2] A. S. Kechris, Ch. Rosendal, *Turbulence amalgamation and generic automorphisms of homogenoeus structures*, ***Proceedings of the London Mathematical Society***, (3) vol. 94 (2007), pp. 302–350.

▸ ONDřEJ JEžIL, *Limits of structures and total NP search problems.*
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For a class of finite graphs, we define a limit object relative to some computationally restricted class of functions. The properties of the limit object then reflect how a computationally restricted viewer "sees" a generic instance from the class. The construction uses Krajíček's forcing with random variables [1]. We prove sufficient conditions for universal and existential sentences to be valid in the limit, provide several examples, and prove that such a limit object can then be expanded to a model of weak arithmetic. We then take the limit of all finite pointed paths to obtain a model of arithmetic where the problem OntoWeakPigeon is total but Leaf (the complete problem for **PPA**) is not. This can be viewed as a logical separation of the oracle classes of total NP search problems, which in our setting implies standard nonreducibility of Leaf to OntoWeakPigeon.

[1] JAN KRAJÍČEK, ***Forcing with random variables and Proof complexity***, Mathematical Society Lecture Note Series, Cambridge University Press, 2011.

▸ ISTVÁN JUHÁSZ, *Resolvability of product spaces and measurable cardinals.*
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All spaces below are crowded (i.e. have no isolated points). A space is κ-*resolvable*, if it can be partitiomed into κ disjoint dense subsets. It is *irresolvable* if it is not 2-resolvable. X is *maximally resolvable* if it is $\Delta(X)$-resolvable, where $\Delta(X)$ is the minimum size of a non-empty open set in X. It is easy to see that the product of infinitely many spaces is always $\mathfrak{c}$-resolvable.

For $n \le \omega$ let $M(n)$ be the statement that there are n measurable cardinals and $\Pi(n)$ ($\Pi^+(n)$) that there are $n+1$ (0-dimensional T_2) spaces whose product is irresolvable. We prove that

1. $M(1)$, $\Pi(1)$ and $\Pi^+(1)$ are equiconsistent;
2. if $1 < n < \omega$ then $CON(M(n))$ implies $CON(\Pi^+(n))$;
3. $CON(M(\omega))$ implies the consistency of having infinitely many 0-dimensional T_2-spaces such that the product of any finitely many of them is irresolvable.

These results settle old problems of Malykhin.

An even older question of Ceder and Pearson asks if the product of a maximally resolvable space with any space is itself maximally resolvable. Concerning this we show that the following are consistent modulo $M(1)$:

1. There is a 0-dimensional T_2 space X with $\omega_2 \le |X| = \Delta(X) \le 2^{\omega_1}$ whose product with any countable space is not ω_2-resolvable.
2. There is a monotonically normal space X with $|X| = \Delta(X) = \aleph_\omega$ whose product with any countable space is not ω_1-resolvable.

These significantly improve a result of Eckertson. Unlike in the first part, here we do not know if large cardinals, or even anything more than ZFC, are needed to get a counterexample to the Ceder-Pearson question.

This is joint work with L. Soukup and Z. Szentmiklóssy.

[1] I. Juhász, L. Soukup, and Z. Szentmiklóssy, *On the resolvability of products*, ***Fundamenta Mathematicae***, vol. 260 (2023), pp. 281–295.

▶ ANNIKA KANCKOS, *On first-order variants of the ontological argument.*
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Gödel's ontological proof of the necessary existence of a godlike object, formally $\Box\exists x.G(x)$, usually requires a system for second-order modal logic in which the possible existence of a godlike object, $\Diamond\exists x.G(x)$, follows through an indirect and therefore classical proof. The implication $\Diamond\exists x.G(x) \to \Box\exists x.G(x)$ is then intuitionistically provable from the axioms and modal principles. However, by turning the ontological axioms and definitions into rules of proof extending the logical calculus, the system can be restricted to a first-order modal logic, thereby making second-order quantification superfluous in multiple standard variants of the argument.

Moreover, the proof of compatibility of positive properties requires classical reasoning, while it is provably impossible to derive it in an intuitionistic system. This can be directly shown by considering an arbitrary intuitionistic derivation of $\exists x.G(x)$ and showing that this entails a proof of $\perp$ in propositional modal logic. Thus, if the logical system is consistent, then $\exists x.G(x)$ is not intuitionistically derivable. The former consistency claim is assumed as trivial, but an easy soundness proof could demonstrate it, with respect to a standard Kripke semantics.

By considering Anderson's emendation [1], a variant by Hájek [3], Scott's variant [4], as well as a minimal axiomatization by Benzmüller [2], we can conclude that the underivability result holds for at least the former two, and the restriction to first-order logic generally holds for variants of the ontological argument.

[1] Anderson, C. A. & Gettings, M. , *Gödel's ontological proof revisited*, In: ***Gödel '96, edited by Hájek P.***, Springer, 1996.

[2] Benzmüller, C. *A Simplified Variant of Gödel's Ontological Argument*. To appear in ***Sophia***, (2022).

[3] Hájek, P. *A new small emendation of Gödel's ontological proof*, ***Studia Logica***, vol. 71 (2002), no. 2, pp. 149–164.

[4] Scott, D., *Appendix B. Notes in Dana Scotts hand.*, In: ***Logic and Theism: Arguments for and against Beliefs in God*** (J. H. Sobel, editor), Cambridge University Press, 2004, p. 145.

▸ DEBORAH KANT, *Predicting axioms.*
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Nowadays, philosophers do not consider mathematical axioms necessarily as self-evident statements. If not self-evident, what are the roles of mathematical axioms in mathematical practice? General ideas by Easwaran [1], Maddy ([3] and [4]), and Schlimm [2] require complementation by analyses of specific uses of axioms in mathematical practice that go beyond the question of axiom adoption. In this talk, I elaborate on the *prediction-use* of large cardinal axioms in set-theoretic practice. The prediction-use of an axiom A consists in a prediction that some statement S that is provable in ZFC$+A$ is probably provable in ZFC only; if such a ZFC-proof can indeed be provided, the prediction is confirmed.

This case study is partially based on information gathered in an interview study with set-theoretic practitioners and augmented by two examples from set-theoretic research: Borel determinacy and Cichoń's maximum. The philosophical appeal of the prediction-use consists in its twofold significance. For one, it is a heuristic use of axioms in the discovery process of mathematical proofs, useful for all set-theoretic practitioners. Secondly, referring to Gödel's ideas on extrinsic justification [5], I argue that each instance of a successful prediction-use provides a verifiable consequence of some axiom, and in this sense, an extrinsic reason in favour of this axiom.

[1] Kenny Easwaran, *The Role of Axioms in Mathematics*, ***Erkenntnis***, vol. 68 (2008), no. 3, pp. 381–391.

[2] Dirk Schlimm, *Axioms in Mathematical Practice*, ***Philosophia Mathematica***, vol. 21 (2013), no. 1, pp. 37–92.

[3] Penelope Maddy, *Believing the Axioms. I*, ***The Journal of Symbolic Logic***, vol. 53 (1988), no. 2, pp. 481–511.

[4] ——— *Believing the Axioms. II*, ***The Journal of Symbolic Logic***, vol. 53 (1988), no. 3, pp. 736–764.

[5] Kurt Gödel, *What is Cantor's Continuum Problem*, ***The American Mathematical Monthly***, vol. 54 (1947), no. 9, pp. 515–525.

▶ EITETSU KEN, *On Σ_0^B-generalizations of counting principles over V^0.*
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Ajtai's discovery ([1]) of $V^0 \nvdash ontoPHP_n^{n+1}$, where $ontoPHP_n^{n+1}$ is a Σ_0^B formalization of the statement "there does not exist a bijection between $(n+1)$ pigeons and n holes," was a significant breakthrough in proof complexity, and there have been many interesting generalizations and variations of this result.

In this talk, we first focus on the following well-known result ([2]): for any $p \geq 2$,

$$V^0 + Count_k^p \nvdash injPHP_n^{n+1},$$

where $Count_k^p$ denotes a Σ_0^B formalization of the modular counting principle mod p and $injPHP_n^{n+1}$ denotes that of the pigeonhole principle for injections.

We try to make this result uniform for p. We give three types of (first-order and propositional) formulae which at first glance seem to be generalized versions of counting principles, and compare their strengths over V^0. In particular, we see two of them, $UCP_n^{l,d}$ and GCP, actually serve as uniform versions of $Count_n^p$ $(p \geq 2)$.

Then we conjecture that $V^0 + UCP_k^{l,d} \nvdash injPHP_n^{n+1}$ and give a sufficient condition to prove it.

[1] Ajtai, M., *The complexity of the Pigeonhole Principle*, ***Combinatorica***, vol.14 (1994), no. X, pp.417–433.

[2] Beame, P., & Riis, S., *More on the relative strength of counting principles*, ***Proof Complexity and Feasible Arithmetics*** (P. Beame, & S. Buss), American Mathematical Society, Providence, RI, 1998, pp.13–35.

▸ ERFAN KHANIKI, *Nisan–Wigderson generators in Proof Complexity: New lower bounds.*
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A map $g : \{0,1\}^n \to \{0,1\}^m$ $(m > n)$ is a hard proof complexity generator for a proof system P iff for every string $b \in \{0,1\}^m \setminus \mathsf{Rng}(g)$, formula $\tau_b(g)$ naturally expressing $b \notin \mathsf{Rng}(g)$ requires superpolynomial size P-proofs. One of the well-studied maps in the theory of proof complexity generators is Nisan–Wigderson generator. Razborov [1] conjectured that if A is a suitable matrix and f is a $\mathsf{NP}\cap\mathsf{CoNP}$ function hard-on-average for $\mathsf{P/poly}$, then $\mathsf{NW}_{f,A}$ is a hard proof complexity generator for Extended Frege.

In this talk, we prove a form of Razborov's conjecture for AC^0-Frege. We show that for any symmetric $\mathsf{NP}\cap\mathsf{CoNP}$ function f that is exponentially hard for depth two AC^0 circuits, $\mathsf{NW}_{f,A}$ is a hard proof complexity generator for AC^0-Frege in a natural setting.

[1] A. A. Razborov, *Pseudorandom generators hard for k-DNF resolution and polynomial calculus resolution*, ***Annals of Mathematics. Second Series***, vol. 191, no. 2, pp. 415–472.

▸ EKATERINA KUBYSHKINA, GIUSEPPE PRIMIERO, *Trustworthy AI: probabilities meet possible worlds.*

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The notion of trustworthiness, central to many fields of human inquiry, has recently attracted the attention of various researchers in logic, computer science and artificial intelligence (AI). Both conceptual and formal approaches for modelling trustworthiness as a (desirable) property of AI systems are emerging in the literature. To develop logics fit for this aim means to analyse both the non-deterministic aspect of AI systems and to offer a formalization of the intended meaning of their trustworthiness. In this work we take a semantic perspective on representing such processes, and provide a measure on possible worlds for evaluating them as trustworthy. In particular, we intend trustworthiness as the correspondence within acceptable limits between a model in which the theoretical probability of a process to produce a given output is expressed and a model in which the frequency of showing such output as established during a relevant number of tests is measured. From a technical perspective, we show that our semantics characterizes the probabilistic typed natural deduction calculus introduced in [1] and further extended in [2]. This contribution connects those results on trustworthy probabilistic processes with the mainstream method in modal logic, thereby facilitating the understanding of this field of research for a larger audience of logicians, as well as setting the stage for an epistemic logic appropriate to the task.

[1] Fabio Aurelio D'Asaro and Giuseppe Primiero, *Probabilistic typed natural deduction for trustworthy computations*, ***Proceedings of the 22nd International Workshop on Trust in Agent Societies (TRUST 2021) Co-located with the 20th International Conferences on Autonomous Agents and Multiagent Systems (AAMAS)*** (London, UK), (Dongxia Wang, Rino Falcone, and Jie Zhang,), vol. 3022, CEUR Workshop Proceedings, 2021.

[2] Fabio Aurelio D'Asaro, Francesco Genco and Giuseppe Primiero, *Checking trustworthiness of probabilistic computations in a typed natural deduction system*, ***CoRR***, abs/2206.12934, 2022.

▸ BEIBUT KULPESHOV, SERGEY SUDOPLATOV, *Almost quite orthogonality of 1-types in weakly o-minimal theories.*

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The present lecture concerns the notion of *weak o-minimality* originally studied by H.D. Macpherson, D. Marker and C. Steinhorn in [1]. A *weakly o-minimal structure* is a linearly ordered structure $M = \langle M, =, <, \dots \rangle$ such that any definable (with parameters) subset of M is a finite union of convex sets in M.

Here we study a new variant of orthogonality of non-algebraic 1-types in weakly o-minimal theories: almost quite orthogonality.

We need the notion of a (p, q)-splitting formula introduced in [2]. Let $A \subseteq M$, $p, q \in S_1(A)$ be non-algebraic, $p \not\perp^w q$. We say that an L_A-formula $\phi(x, y)$ is a *(p, q)-splitting formula* if there exists $a \in p(M)$ such that $\phi(a, M) \cap q(M) \neq \emptyset$, $\neg\phi(a, M) \cap q(M) \neq \emptyset$, $\phi(a, M) \cap q(M)$ is convex and $\inf[\phi(a, M) \cap q(M)] = \inf q(M)$.

Let T be a weakly o-minimal theory, $M \models T$, $A \subseteq M$, $p, q \in S_1(A)$ be non-algebraic. We say that p is not *almost quite orthogonal* to q if there exist a (p, q)-splitting formula $\phi(x, y)$ and an A-definable equivalence relation $E_q(x, y)$ partitioning $q(M)$ into infinitely many convex classes so that for any $a \in p(M)$ there is $b \in q(M)$ such that $\sup \phi(a, M) = \sup E_q(b, M)$. We say that T is *almost quite o-minimal* if the notions of weak and almost quite orthogonality of 1-types coincide.

Theorem 1. *Let T be a weakly o-minimal theory of finite convexity rank having less than 2^ω countable models, $\Gamma_1 = \{p_1, p_2, \dots, p_m\}$, $\Gamma_2 = \{q_1, q_2, \dots, q_l\}$ be maximal pairwise weakly orthogonal families of quasirational and irrational 1-types over $\emptyset$ respectively for some $m, l < \omega$. Then T has exactly $3^m 6^l$ countable models iff T is almost quite o-minimal.*

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[1] H.D. Macpherson, D. Marker, and C. Steinhorn, *Weakly o-minimal structures and real closed fields*, ***Transactions of the American Mathematical Society***, Vol. 352, No. 12 (2000), pp. 5435–5483.

[2] B.Sh. Kulpeshov, *Criterion for binarity of $\aleph_0$-categorical weakly o-minimal theories*, ***Annals of Pure and Applied Logic***, Vol. 45 (2007), pp. 354–367.

▸ BEIBUT KULPESHOV, *On algebras of binary formulas for weakly circularly minimal theories with a trivial definable closure.*
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Algebras of binary formulas are a tool for describing relationships between elements of the sets of realizations of 1-types at binary level with respect to superpositions of binary definable sets. We consider algebras of binary isolating formulas originally studied in [1, 2], where under a binary isolating formula we understand a formula of the form $\varphi(x, y)$, without parameters, such that for some parameter a the formula $\varphi(a, y)$ isolates some complete type from $S_1(\{a\})$.

The notion of *weak circular minimality* was originally studied in [3]. A *weakly circularly minimal structure* is a circularly ordered structure $M = \langle M, K_3, \dots \rangle$ such that any definable (with parameters) subset of M is a union of finitely many convex sets in M. In [4] $\aleph_0$-categorical 1-transitive non-primitive weakly circularly minimal structures of convexity rank 1 with a trivial definable closure have been described up to binarity.

Here we discuss algebras of binary isolating formulas for these structures and give the following criterion for commutability of such algebras:

THEOREM 1. *Let M be an $\aleph_0$-categorical 1-transitive non-primitive weakly circularly minimal structure of convexity rank 1 with $\mathrm{dcl}(a) = \{a\}$ for some $a \in M$. Then the algebra $\mathfrak{P}_M$ of binary isolating formulas is commutable iff for any convex-to-right formula $R(x, y)$ that is not equivalence-generating the function $r(y) := \mathrm{rend}\, R(M, y)$ is monotonic-to-right on M.*

This research has been funded by Science Committee of Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. BR20281002).

[1] S.V. SUDOPLATOV, ***Classification of countable models of complete theories***, Novosibirsk, Edition of NSTU, 2018.

[2] I.V. SHULEPOV, S.V. SUDOPLATOV, *Algebras of distributions for isolating formulas of a complete theory*, ***Siberian Electronic Mathematical Reports***, Vol. 11 (2014), pp. 362–389.

[3] B.SH. KULPESHOV, H.D. MACPHERSON, *Minimality conditions on circularly ordered structures*, ***Mathematical Logic Quarterly***, Vol. 51, No. 4 (2005), pp. 377–399.

[4] B.SH. KULPESHOV, *On $\aleph_0$-categorical weakly circularly minimal structures*, ***Mathematical Logic Quarterly***, Vol. 52, No. 6 (2006), pp. 555–574.

▸ LEONARD KUPŚ AND ALEXANDER BOLOTOV, *Hypersequent calculus for PLTL*.

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Linear Time Logic (LTL) has been considered in a number of proof systems, ranging from tableaux methods, natural deduction and sequent calculi. The temporal reasoning present in sequent calculi and tableaux methods is similar and faces similar obstacles, thus it is possible to draw analogies between the two systems. However, it cannot be said about labelled natural deduction [1] and sequent calculi. To fill this gap, we offer a hypersequent calculus for PLTL based on reasoning from labelled natural deduction.

[1] Bolotov, A., Grigoriev, O. and Shangin, V., *A Simpler formulation of natural deduction calculus for linear-time temporal logic*, ***Proceedings of the 3rd Indian International Conference on Artificial Intelligence*** (Pune, India), (P. Bhanu, editor), 2007, pp. 1253-1266.

▸ MATEUSZ ŁEŁYK, *On sets definable via pathologies in satisfaction classes.*
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A *satisfaction class* in a nonstandard model $\mathcal{M}$ of Peano Arithmetic (PA) is essentially a satisfaction relation which validates the usual compositional Tarski's conditions for all formulae *in the sense of* $\mathcal{M}$. While a classical theorem, originally due to Kotlarski, Krajewski and Lachlan, shows that every countable and recursively saturated model of PA carries a satisfaction class, recent developments provided many limitative results on how well-behaved can such a satisfaction class be. In particular, it was shown in [1], that unless $\mathcal{M}$ satisfies a proper and rather strong extension of PA (all finite iterations of uniform reflection over PA), every satisfaction class S on $\mathcal{M}$ exhibits the following pathology: there is a disjunction with a nonstandard number of disjuncts which is true (according to S) but whose each disjunct is deemed false (by S). Somewhat surprisingly, in an arbitrary countable and recursively saturated model $\mathcal{M}$ of PA one can find a satisfaction class S which behaves correctly on arbitrarily long disjunctions in the following sense: every disjunction with a true (according to S) disjunct is itself true (according to S).

In the talk we scrutinize this picture and study various variants of the following general question: when for a subset X of a countable recursively saturated model $\mathcal{M}$ of PA one can find a satisfaction class S such that X is the set of lengths of those disjunctions on which S behaves correctly? We solve this problem completely for the special case of idempotent disjunctions, i.e. the ones in which every disjunct is the same formula (but repeated many times). Firstly, we show that for a set $X \subseteq \mathcal{M}$, satisfying some minimal obvious conditions, there is a satisfaction class S such that X is precisely the set of lengths of those idempotent disjunctions of the sentence $0 = 1$ which are false (according to S) if and only if X satisfies a weak saturation condition we call *separability*. As a consequence, we show that in a model $\mathcal{M}$ the standard cut ω can be defined via a satisfaction class in this way if and only if $\mathcal{M}$ is arithmetically saturated. Secondly, we show when for a cut $I \subseteq \mathcal{M}$ there is a satisfaction class S such that I consists of those numbers c such that S behaves correctly on *every* idempotent disjunction of length $\leq c$. We show that in a (countable and recursively saturated) model $\mathcal{M}$ every cut can be characterised in this way if and only if $\mathcal{M}$ is arithmetically saturated. Finally we show how to generalize these results to various other propositional operations like taking idempotent conjunctions and adding long blocks of double negations.

This is joint work with Athar Abdul-Quader which builds upon an unpublished work due to James Schmerl.

[1] Cezary Cieśliński, Mateusz Łełyk, Bartosz Wcisło, *The Two Halves of Disjunctive Correctness*, ***Journal of Mathematical Logic***, https://doi.org/10.1142/S021906132250026X

▸ DOROTA LESZCZYŃSKA-JASION, *A sequent system for a Boolean non-Fregean logic WB*.

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Logic WB is a Boolean non-Fregean logic introduced to the literature by Roman Suszko [2]. The idea of non-Fregean logics (NFL) stems from Wittgenstein's *Tractatus*, more specifically—the semantics and ontology as suggested by this work [1, 3]. Formally, an NFL is built upon classical propositional logic CPL by adding the identity connective $\equiv$ to the language. Intuitively, '$\alpha \equiv \beta$' is used to express the fact that α and β describe the same situation. The basic NFL proposed by Suszko, sentential calculus with identity (SCI), has a drawback (at least, one may view it as such): hardly anything can be stated about identity of situations in this logic—all SCI-valid equations are of the form '$\alpha \equiv \alpha$'.

WB is one of NFLs strengthening SCI by allowing $\equiv$ to have some Boolean properties; for example, '$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$' is a validity in WB. Still, $\equiv$ in WB is not truth-functional equivalence.

A little is known about proof-theoretical properties of this logic—the original account is axiomatic. In the talk I present a sequent system for WB (based on idea by Agata Tomczyk) together with a proof procedure by means of which positive decidability of WB is shown. I also introduce a new semantics of truth valuations for WB (as far, only algebraic semantics was available).

[1] Roman Suszko, *Ontologia w Traktacie L. Wittgensteina (Ontology in the Tractatus of L. Wittgenstein)*, ***Studia Filozoficzne***, vol. 1 (1968), pp. 97–121.

[2] Roman Suszko, *Identity connective and modality*, ***Studia Logica***, vol. 27 (1971), pp. 7–39.

[3] Roman Suszko, *Abolition of the Fregean axiom*, ***Logic Colloquium*** (Boston 1972–1973), (R. Parikh, editor), vol. 453 of *Lecture Notes in Mathematics*, Springer Verlag, 1975, pp. 169–239.

▸ LAURENŢIU LEUŞTEAN, PEDRO PINTO, *Proof mining and asymptotic regularity*.
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This talk presents a recent application of proof mining to the asymptotic behavior of the alternating Hapern-Mann iteration for nonexpansive mappings [2]. Proof mining is a subfield of applied proof theory concerned with the extraction of new quantitative and qualitative information from mathematical proofs, with the help of proof-theoretic tools. This paradigm of research, developed by Ulrich Kohlenbach and collaborators, is inspired by Kreisel's program on unwinding of proofs from the 1950s. We present extensions to UCW-hyperbolic spaces of the quantitative asymptotic regularity results for the alternating Halpern-Mann iteration obtained by Dinis and Pinto for CAT(0) spaces [1]. These results are new even for uniformly convex normed spaces. Furthermore, for a particular choice of the parameter sequences, we compute linear rates of asymptotic regularity in W-hyperbolic spaces and quadratic rates of T- and U-asymptotic regularity in CAT(0) spaces.

[1] B. Dinis, P. Pinto, *Strong convergence for the alternating Halpern-Mann iteration in CAT(0) spaces*, arXiv:2112.14525 [math.FA]; accepted for publication in ***SIAM Journal on Optimization*** (2023).

[2] L. Leuştean, P. Pinto, *Rates of asymptotic regularity for the alternating Halpern-Mann iteration*, arXiv:2206.02226 [math.OC]; accepted for publication in ***Optimization Letters*** (2023).

▶ TALIA LEVEN, *Robinson's diagramas a tool for dealing with Skolem's criticism of formal language.*

Robinson's diagramas a tool for dealing with Skolem's criticism of formal language abstract In this paper, I wish to present Robinson's diagram as an attempt to overcome the problem of formal language, which was indicated in 1922 by Skölem as the "relativity of set-theoretical notions". Robinson's diagram is a symbolic representation of information. The diagram of a mathematical structure M is the set of all elementary sentences of one of the forms φ or $\neg\varphi$ which hold in M, where $\varphi = R(a_1, \ldots a_n)$ for any $R, a_1, \ldots, a_n$ which denotes relations of individuals $a1, \ldots, an$, in M. The diagram is syntactic and semantic at the same time. Using the diagram with the philosophical position that links semantics and syntactic it is possible to find the unique model, that is described by a set of axioms.

▸ STEPHEN MACKERETH AND JEREMY AVIGAD, *Two-sorted Frege Arithmetic is not conservative.*
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Neo-Fregean logicists Hale and Wright [1] have claimed that Hume's Principle (HP) may be taken as an implicit, stipulative definition of cardinal number, true simply by fiat. A longstanding problem for neo-Fregean logicism is that HP is not deductively conservative over the theory to which it is added, namely, pure axiomatic second-order logic. This seems to preclude HP from being true by fiat. In this talk, we study Richard Kimberly Heck's [2] theory of Two-sorted Frege Arithmetic (2FA), a variation on HP which has been thought to be deductively conservative over second-order logic. We show that it isn't. In fact, 2FA is not conservative over n-th order logic, for all $n \geq 2$. It follows that in the usual one-sorted setting, HP is not deductively Field-conservative (in the sense of Weir [3]) over second- or higher-order logic.

[1] Bob Hale and Crispin Wright, ***The Reason's Proper Study: Essays towards a Neo-Fregean Philosophy of Mathematics***, Clarendon Press, Oxford, 2001.

[2] Richard Kimberly Heck, *The Julius Caesar objection*, ***Language, Thought, and Logic: Essays in Honour of Michael Dummett*** (Richard Kimberly Heck, editor), Oxford University Press, Oxford, 1997, pp. 273–308.

[3] Alan Weir, *Neo-Fregeanism: An embarrassment of riches*, ***Notre Dame Journal of Formal Logic***, vol. 44 (2003), no. 1, pp. 13–48.

► DAVIDE MANCA, *On a weak notion of well order*.
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The subsystem $\mathbf{ATR}_0$ is a natural environment for the study of countable well orders in the framework of reverse mathematics. In this talk, we consider an alternative notion of well order, defined in terms of embeddings between initial segments. We call the countable linear orders that satisfy the alternative definition weak well orders. The fact that all countable well orders are weak well orders is provable even in very weak subsystems. On the other hand, the reverse implication follows from Laver's theorem. We show that $\mathbf{ATR}_0$ is equivalent to the fact that all weak well orders are well orders.

The results presented consist of joint work with A. Freund.

▸ ALBERTO MARCONE, *Jumping in the Weihrauch degrees.*
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The jump is a fundamental operation in the Turing degrees and jumps operators have been defined also in other degree structures, such as the enumeration degrees.

Given a quasi-order $(Q, \leq)$, an operator $J : Q \to Q$ can be considered a jump if it satisfies $p < J(p)$ and $J(p) \leq J(q)$ whenever $p \leq q$ (the latter condition ensures that J is degree-theoretic, i.e. can be lifted to the quotient partial order).

In the Weihrauch degrees a natural operator called "jump" was introduced a few years ago ([1]) and then widely used. However this operator fails to satisfy both abstract properties mentioned above (although it satisfies the second one with respect to strong Weihrauch reducibility). We propose a natural definition of a jump operator which satisfies both properties and we compute explicitly the jumps of many well-known Weihrauch degrees. This jump is connected with the (non degree-theoretic) operation of total continuation.

If time allows, we will also mention results about the existence of jumps in arbitrary quasi-orders.

This is joint work with Uri Andrews, Steffen Lempp, Joe Miller and Manlio Valenti.

[1] Brattka, Vasco; Gherardi, Guido; Marcone, Alberto, *The Bolzano-Weierstrass theorem is the jump of weak Kőnig's lemma*, ***Annals of Pure and Applied Logic***, vol. 163 (2012), no. 6, pp. 623–655.

▸ ROBIN MARTINOT AND FRANCESCA POGGIOLESI, *Purity and explanatoriness of proof*.

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Philosophy of mathematics distinguishes several qualities or ideals of proof, and in this talk we will focus on two such ideals. First, 'purity' of proof restricts the methods of a proof to those that in some sense intrinsically belong to a theorem. Here, notions that are thought extraneous to a theorem are excluded from a proof. Second, 'explanatoriness' of proof is the property that a proof not only convinces us that a theorem is true, but additionally shows us *why* it holds.

We aim to investigate the interaction between purity and explanatoriness. From [2] and [4], we gather that Bolzano considered purity and explanatoriness to go together. It seems natural that purity provides an explanation by providing the true reasons that a theorem is true, while impurity raises the question where the extraneousness comes from. On the other hand, a specific kind of explanation is also thought to go together with impurity of proof: impurity unifies results and shows previously unnoticed connections, and "in such a case, the more general context has greater explanatory power" [3]. Additionally, there are suggestions that "a proof's explanatory power is independent of its purity" [5].

We will study the relation between purity and explanatoriness further by considering particular notions of purity (e.g., 'topical' purity [1]) and explanatoriness (e.g., 'conceptual' explanatoriness [7]), and analyzing their behaviour for various proofs of theorems such as the Infinitude of Primes and Pythagoras' Theorem. A broader analysis of the behaviour of these notions will give us more insight into the ideals of purity and explanation themselves, and the differences between such ideals.

Finally, there has been some research on purity and explanatoriness in the context of formal (syntactic) derivations (see e.g. [6]). We will conclude by giving an expectation of the interaction of these properties in the formal setting.

[1] Arana, Andrew and Detlefsen, Michael, *Purity of methods*, ***Philosophers' Imprint***, vol. 11 (2011), no. 2, pp. 1–20.

[2] Baracco, Flavio, *Explanatory Proofs in Mathematics*, ***Logique et Analyse***, (2017), no. 237, pp. 67–86.

[3] Dawson, John W, *Why do mathematicians re-prove theorems?*, ***Philosophia Mathematica***, vol. 14 (2006), no. 3, pp. 269–286.

[4] Franks, Curtis, *Logical completeness, form, and content: an archaeology*, ***Interpreting Gödel: Critical Essays***, (2014), pp. 78–106.

[5] Lange, Marc, *Explanation, Existence and Natural Properties in Mathematics–A Case Study: Desargues' Theorem*, ***Dialectica***, vol. 69 (2015), no. 4, pp. 435–472.

[6] Poggiolesi, Francesca, *On constructing a logic for the notion of complete and immediate formal grounding*, ***Synthese***, vol. 195 (2018), pp. 1231–1254.

[7] Poggiolesi, Francesca and Genco, Francesco, *Conceptual (and hence mathematical) explanation, conceptual grounding and proof*, ***Erkenntnis***, (2021), pp. 1–27.

▸ FABIO MASSAIOLI, *Non-trivial invariants of rule permutation and cut-elimination in classical sequent calculus.*

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Cut-elimination procedures in classical sequent calculus are notoriously non-deterministic and non-confluent, both in the original formulation by Gentzen and in later reformulations. It is natural to ask whether those instances of non-confluence are superficial in nature, i.e. whether distinct normal forms of the same derivation are in fact correlated in a non-trivial way. A famous counter-example by Lafont purports to show that the answer is negative, that is, any notion of proof equivalence compatible with classical cut-elimination must be a trivial one that identifies all proofs of the same sequent. A long standing open question has been whether it is possible to work around Lafont's example by natural and non-trivial adjustments of the calculus and/or of cut-reduction steps, without resorting to symmetry-breaking techniques like polarization or embeddings into intuitionistic or linear logic.

Working within the propositional fragment of GS4 – i.e. the one-sided variant of Kleene's context-sharing style sequent system G4 – where parallel rule applications permute freely, we show that the graph induced by axiom rules linking dual atom occurrences is preserved under arbitrary rule permutations in cut-free proofs. We then refine the notion of axiom-induced graph so as to extend the result to proofs with cuts. Finally, we exploit rule permutations to define a global normalization procedure that preserves axiom-induced graphs, thus yielding a non-trivial invariant of cut-elimination in GS4.

We conclude by discussing briefly the possibility of extending the result to context-splitting style systems and first-order logic.

► GUILLAUME MASSAS, *Duality for fundamental logic.*
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Holliday [2] recently introduced a non-classical logic called *fundamental logic*, which is meant to capture only those properties of the connectives $\wedge, \vee$ and $\neg$ that hold in virtue of their introduction and elimination rules in Fitch's natural deduction system for propositional logic. Holliday provides a semantics for fundamental logic in terms of *compatibility frames* (sets endowed with a relation of compatibility between its points) which generalizes both Goldblatt's semantics for orthologic and Kripke semantics for intuitionistic logic. In particular, any relation R on a set X determines a closure operator on $\mathcal{P}(X)$, and Holliday shows that any lattice can be represented as a sublattice of the fixpoints of such a closure operator for some compatibility frame (X, R).

In this talk, I will show how standard tools from duality theory allow one to lift Holliday's representation theorem to a full duality between the category of lattices and a category of topologized compatibility frames. The key idea is to embed any lattice into the fixpoints of a Galois connection on a distributive lattice in order to then use a version of the duality between modal distributive lattices and coalgebras of the Vietoris functor on the category of Priestley spaces [1, 3]. Time permitting, I will also show how this duality yields natural semantics for any extension of fundamental logic with connectives that are monotone (i.e., ordering-preserving or order-reversing) in each coordinate.

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[2] Holliday, Wesley H., *A fundamental non-classical logic*, ***arXiv preprint arXiv:2207.06993***, (2022).

[3] Palmigiano, Alessandra, *A coalgebraic view on positive modal logic*, ***Theoretical Computer Science***, vol. 327 (2004), pp. 175–195.

▶ BRETT MCLEAN, *Complete representation by partial functions for signatures containing antidomain restriction*.

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In [2], Jackson and Stokes investigate the axiomatisability of classes of algebras that are representable as (i.e. isomorphic to) an algebra of partial functions. Using a uniform method of representation, they give, for around 30 different signatures containing the *domain restriction* operation, either a finite equational or finite quasi-equational axiomatisation of the class of representable algebras. Only a handful of these classes had previously been axiomatised.

We show that a similar uniform method of representation can be used to characterise many of the corresponding subclasses of completely representable algebras. A complete representation is one that turns any existing infima/suprema into intersections/unions. Specifically, we do this for signatures containing the operation called *minus* in [2] and which we call *antidomain restriction*; thus for about half of the signatures treated in [2]. Together with the results of [2], this gives us finite first-order axiomatisations of these classes of completely representable algebras. Only a couple of complete representation classes had previously been axiomatised (for representation as partial functions) [3, 1].

[1] Célia Borlido and Brett McLean, *Difference–restriction algebras of partial functions: axiomatisations and representations*, ***Algebra Universalis***, vol. 83 (2022), no. 3, 27 pp.

[2] Marcel Jackson and Tim Stokes, *Restriction in Program Algebra*, ***Logic Journal of the IGPL***, (2022), 35 pp.

[3] Brett McLean, *Complete representation by partial functions for composition, intersection and antidomain*, ***Journal of Logic and Computation***, vol. 27 (2017), no. 4, pp. 1143–1156.

▸ RUSSELL MILLER, *Skolem functions and definable subsets of the absolute Galois group of* $\mathbb{Q}$.

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Fix a computable presentation $\overline{\mathbb{Q}}$ of the algebraic closure of the field $\mathbb{Q}$ of rational numbers. With such a presentation, the automorphisms of $\overline{\mathbb{Q}}$ are naturally given as paths through a strongly computable finite-branching tree. The operations of composition and inversion on these automorphisms (i.e., on these paths) are both type-2 computable. Thus we have an effective way of considering $\mathrm{Aut}(\overline{\mathbb{Q}})$, the absolute Galois group of $\mathbb{Q}$.

In this context, one can discuss the computability of Skolem functions for $\mathrm{Aut}(\overline{\mathbb{Q}})$. We show that for *positive formulas* (not using the negation connective) with parameters, Skolem functions are close to computable: one can compute an approximation to the jump of a witness to an existential formula. (That is, these Skolem functions are *low*, in the sense of Brattka, de Brecht, and Pauly.) The same holds for Skolem functions for any Π_2 formula, positive or not, and for certain larger classes of formulas as well. We also present related results describing the subsets of $(\mathrm{Aut}(\overline{\mathbb{Q}}))^n$ definable by such formulas.

▶ OWEN MILNER, *Formalizing the Whitehead tower in cubical agda.*
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This talk will present details of a formalization, in cubical agda, of the key properties of the Whitehead tower. This construction has been an important tool for computing the algebraic invariants of spaces since the work of Cartan and Serre [1] and Whitehead [2] in the early 1950s. The recent development of homotopy type theory (as in [3], and [4]) has made it possible for significant parts of classical algebraic topology to be developed synthetically and constructively, and in a manner suitable for computer formalization. Work such as that being presented here connects a canonical part of pure mathematics with the burgeoning interest in formalization and verification of mathematics by computers. The formalization includes not only the definition of the Whitehead tower, but also a proof of that its objects satisfy a universal property, the computation of their homotopy groups, and the identification of the fibers of the structure maps of the tower as particular Eilenberg-MacLane spaces. Parts of the formalization are available online [5].

[1] Cartan, Henri and Serre, Jean-Pierre, *Espaces Fibrés et Groupes d'Homotopie, I*, ***Comptes Redus Hebdomadaires de Séances de l'Académie des Sciences*** Académie de Sciences (France), Quai des Grands-Augustins, 55, Paris, France, 1952, pp. 288–290.

[2] Whitehead, George W., *Fiber Spaces and the Eilenberg Homology Groups*, ***Proceedings of the National Academy of Sciences 38(5)*** (Linus Pauling, Edwin B. Wilson et al., editors), National Academy of Sciences of the United States of America, 2101 Constitution Avenue, Washington 25, D. C., USA, 1952, pp. 426–430.

[3] The Univalent Foundations Program, ***Homotopy Type Theory: The Univalent Foundations of Mathematics***, https://homotopytypetheory.org/book/, The Institute for Advanced Study, 2013.

[4] Cohen, Cyril and Coquand, Thierry and Huber, Simon and Mörtberg, Anders, *Cubical Type Theory: A Constructive Interpretation of the Univalence Axiom*, ***Post-Proceedings of the 21st International Conference on Types for Proofs and Programs (TYPES 2015)*** (Tallinn, Estonia), (Tarmo Uustalu, editor), Dagstuhl Publishing, 2018, pp. 129–162.

[5] Milner, Owen and Barton, Reid and Ljungström, Axel et al. *Code Repository*, https://github.com/CMU-HoTT/serre-finiteness

▸ MOJTABA MOJTAHEDI, *On provability logic of Heyting Arithmetic*.
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We axiomatize the provability logic for Heyting Arithmetic and show that it is decidable. Mainly we focus on the two handful tools: (1) a relativised version of unification named NNIL-fication, (2) Hybrid semantics for provability logics.

▸ TOMMASO MORASCHINI, JOHANN J. WANNENBURG, KENTARO YAMAMOTO,
Elementary equivalence in positive logic via prime products.
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A generalization of ultraproducts called prime products will be introduced. While ultraproducts preserve sentences of first-order logic (Łoś), prime products preserve sentences of positive logic in the sense of Poizat. The main result is an analogue of the Keisler-Shelah Theorem: two structures in the same language have the same positive theory if and only if some prime product of ultrapowers of one is isomorphic to some prime product of ultrapowers of the other.

▸ JOACHIM MUELLER-THEYS, *A Mathematical Model of the Atom.*
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Following the idea of Wilfried Buchholz, we model the fundamental concept of atomic physics and chemistry by *3-tuples*

$$A := (P, N; E),$$

whereby $P \neq \emptyset$, N, E be *finite sets* of protons, neutrons, electrons respectively. Accordingly, (P, N) models the *nucleus* of A.

In the next step, *functions* assign the numbers of protons, neutrons, electrons:

$$\pi(A) := |P|,\ \nu(A) := |N|,\ \varepsilon(A) := |E|.$$

The definitions for *(atomic) ions* now arise immediately—not requiring higher conceptuality. For example, A is an *anion* if $\varepsilon(A) > \pi(A)$.

The proton number function induces the *equi-valence*

$$A \simeq_\pi B :\Longleftrightarrow \pi(A) = \pi(B).$$

Cf. "Equivalence", The Bulletin of Symbolic Logic 28 (2022), pp. 564-5 (with the misprint $\bigcup \mathcal{P}$).

Now the *equivalence classes*

$$A/\pi := A/\simeq_\pi := \{B\colon B \simeq_\pi A\},$$

constituting a partition of the atoms, model the *(chemical) elements*. Each element A/π is characterized by $\pi(A)$. Eventually, these *order numbers* may be assigned to names and symbols, like hydrogen, $\mathrm{H} \mapsto 1$. (*Compare* "A Mathematical Linguistics", BSL 24 (2018), pp. 114-5 (ASL@JMM2017 (Atlanta))).

Moreover, the equi-valence induced by the number of *neutrons* partitions each element into its *isotopes*, and underlying *systems of atoms* $\mathcal{A}$ ought to be further specified.

Acknowledgment. Ongoing thanks to 'Peana Pesen' and Heike & Andreas Haltenhoff. The author wants to mention following chemists: O. Mueller, M. R. Bloch, John T. Wasson, Walter Littke, Wolfgang Maier-Borst, Stefan Reimann-Andersen, Bernhard Höferth, Claudia Friesen, Jerry LR Chandler.

▸ M.A. NAIT ABDALLAH, *Quantum as logic: on the logic resolution of the three-polarizer paradox in quantum mechanics.*
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Polarization of light is a quantum phenomenon which is directly accessible to our senses. It also gives rise to the three-polarizer paradox in quantum mechanics.

We present a constructive logic solution to this paradox, inspired from Curry-Howard correspondence [3]. This quantum constructive logic differs from quantum logics in the style of Birkhoff-von Neumann [1].

Formulae are defined by the following formal grammar, where $\sqcup$ is a non-Boolean formula constructor corresponding to quantum physical state superposition, F stands for falsehood, and elements of $\mathcal{B}$ are distinct

$$\begin{array}{ll} \mathcal{F} ::= \mathcal{F}_c \mid \mathcal{Q} & \mathcal{F}_c ::= \mathsf{F} \mid \mathcal{E} \mid (\mathcal{E} \to \mathsf{F}) \\ \mathcal{Q} ::= \sqcup \mathcal{B} & \mathcal{B} ::= \{\mathcal{E}, \mathcal{E}\} \\ \mathcal{E} ::= \sigma_1 \mid \sigma_2 \mid \ldots & \end{array}$$

The domain of proof terms decomposes into three categories: set $\boldsymbol{T}$ of *quantum transition terms* (or τ*-terms*), $\boldsymbol{A}$ of *amplitude terms* and $\boldsymbol{B}$ of *base terms*, defined by the following formal grammar

$$\begin{aligned} \boldsymbol{T} &::= (\!|\boldsymbol{A}, \boldsymbol{A}|\!) \mid \widehat{\delta}_{\mathbf{I}}\boldsymbol{A} \mid \boldsymbol{T}^{\mathbf{W}} \mid (:\boldsymbol{T}/\boldsymbol{T}) \mid (:\widehat{\delta}_{\mathbf{I}}/\boldsymbol{T}) \mid \boldsymbol{\omega}\left(\frac{\boldsymbol{T}}{\boldsymbol{T}}\right) \\ \boldsymbol{A} &::= \mathbf{0} \mid \boldsymbol{B} \\ \boldsymbol{B} &::= V \mid \boldsymbol{B}^{\mathbf{W}} \mid (\boldsymbol{B}\boldsymbol{B}) \mid \boldsymbol{\mu}\left(\frac{\widehat{\delta}_{\mathbf{I}}}{\boldsymbol{T}}\right) \mid \bar{\boldsymbol{\mu}}\left(\frac{\widehat{\delta}_{\mathbf{I}}}{\widehat{\delta}_{\mathbf{I}}\boldsymbol{A}}\right) \\ V &::= x \mid y \mid z \mid \ldots \\ \mathbf{W} &::= [0,1] \subseteq \mathbb{R} \\ \mathbf{I} &::= 1 \mid 2 \end{aligned}$$

The three-polarizer paradox experiments are then expressed by context

$$\Gamma = \{x : v, \left(\frac{:\widehat{\delta_1}}{\widehat{\delta_1}x}\right) : v \sqcup h, \left(\frac{:\widehat{\delta_2}}{\widehat{\delta_1}x}\right) : v \sqcup h, \left(\frac{:\widehat{\delta_2}}{(:(\!|z, z|\!)/\widehat{\delta_1}x)}\right) : v \sqcup h\}$$

where v (resp. h) stands for the photon being in the vertical (resp. horizontal) linear polarization state. One then shows that this quantum logic framework, with the corresponding τ-calculus on proof terms and system of natural deduction inference rules, yields a solution to the three-polarizer paradox, provided that some specific form of Bohr complementarity principle [2] is embedded in the structure of the new constructive quantum logic. The embedding uses a notion of *critical pair* of judgments and replaces the notion of theoremhood with membership in some *quantum extension* E of the initially given data, where $E = \bigcup_{n=0}^{\infty} E_n$ with sequence of sets $(E_n)_{n\in\mathbb{N}}$ defined by $E_0 = \Gamma_0$ and

$$\begin{aligned} E_{n+1} = Th(E_n) \quad &\cup \quad \{M : \varphi_i \mid \Gamma \vdash M : \varphi_i \text{ and } E \text{ contains no critical pair}\} \\ &\cup \quad \{N : \neg\varphi_i \mid \Gamma \vdash N : (\varphi_i \to \mathsf{F}) \text{ and } E \text{ contains no critical pair}\}. \end{aligned}$$

[1] G. Birkhoff and J. von Neumann, *The logic of quantum mechanics*, ***Annals of Mathematics***, vol. 37 (1936), pp.823–843.

[2] N. Bohr, ***Atomic theory and the description of nature***, pp.10, 52–91, Cambridge University Press, 1961.

[3] M.H.B. Sørensen and P. Urzyczyn, ***Lectures on the Curry-Howard isomorphism***, Elsevier, 2006.

▶ MYKYTA NARUSEVYCH, *Models of Bounded Arithmetic and variants of the Pigeonhole Principle.*

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We show that the bijective pigeonhole principle $ontoPHP(R)$ is not provable in the theory $T^1_2(R)$ augmented by instances of the weak pigeonhole principle for all $\Delta^b_1(R)$-formulas. The result is achieved by using forcing technique together with a new combinatorial argument that allows us to bypass the switching lemma. As a by-product we construct a model of $T^1_2(R)$ where the usual pigeonhole principle stated for all $\Delta^b_1(R)$-formulas holds up to $n^{1-\epsilon}$ but not for n, where n is a non-standard number and ϵ is arbitrarily small but standard. This can be seen as a step towards an open problem posed by M. Ajtai [1].

[1] M. Ajtai, *Parity and the Pigeonhole Principle*, ***Feasible Mathematics, Progress in Computer Science and Applied Logic***, (S.R. Buss and P. Scott, editors), Birkhäuser, Boston, 1990, pp. 1–24.

► CYRUS F NOURANI, PATRIK EKLUND, *Ultrafilters on n-types categories and the V Universe*.
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Let us start with n-types and positive local realizability (Keiser 1971, Nourani 2003): Given a theory T and a nonnegative integer n, let n(T) be the set of all signature $(\phi) \subseteq signature(T)$.

Let us consider (a) An ordinary category on T-Sigma trees for a signature Sigma and Sigma homomorphisms. (b) Category of direct product models realizing an n-Type: e.g. Horn filters (Nourani 2007). c. Term functors direct "product algebra" category. Objects are term functors and morphims are natural trasnformations on representation preschievs (Nourani 2006). Call these nD-type embedding categories, or F-Type categories. Theorem 1 There is a generic functor on the category the omitting n-types realizing a direct product model (Nourani 2016). From Nourani 2015 volume: algebraic set theory: \\V onto the Boolean models, \\VB, e.g. Scott models for classes of Boolean algebras can be reduced to only the Boolean algebras over $\{0, 1\}$. Proposition (Nurani 2005) stipulations on \\V and be carried on \\VB applying generic definable diagrams on set models, e.g. Gödel operations definable. From Kiesler 70's: Theorem 2 Let T be a countable theory. For each $i \in \omega$, let pi be an essentially nonprincipal n-ype over T. Then T has a model which omits pi for each $i \in \omega$. Let P(T Sig) be a functor category defined on the free signature trees with the power set on T Sig, this can be a monoidal category. The adjunction functors being the functor F, forgetful to the product signature n-type category with G the embedding functor from the n-type category on the product pair signature to a powerset category. Lemma F.G is a Monad on pair product signature n-type. Theorem 3 There are embedding functors from F-Type to the direct product category realizing a filter for the product algebra trees on nD-types. (Nourani-Eklund 2016 MAA and,AAA Vienna2018)

[1] Nourani-Eklund 2017: Term Functors, Ultrafilter Categorial Computing, and Monads Cyrus F Nourani and Patrik Eklund: Coauthors. LAMBERT Academic Publishing Bahnhofstraße 28, D-66111 Saarbrücken, Germany).

▸ FAUSTINE OLIVA, *How can hypermaps guide us through the computer-assisted proof of the Four Colour Theorem?*.

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The Four Colour Theorem's proof [1] [4] is the first substantial result which has been entirely formalized and checked by the proof-assistant Coq [3]. Coq enables the user to write in one programming language — the Calculus of Inductive Constructions — both the mathematical argument and the computational part of the proof. The correctness of the proof-as-program entails the validity of the proof. Because it is hard for a computer to deal with topological arguments, the original statement of the theorem which is about maps had to be rephrased: it became a statement relying on hypermaps. Traditionally, a hypermap is a combinatorial structure defined by a couple of permutations on a finite set [2]. Our hypothesis is that understanding why the hypermap is suitable to rephrase the original problem and prove the theorem in Coq and how its properties are used in order to do so sheds a light on the design of this computer-assisted proof. First, we will show how and why we move from the topological field to the combinatorial one. Secondly, we will present how the hypermap is implemented and used as a data structure. Finally, we will focus on the structural and teleological properties of the proof that had been highlighten.

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▸ GITI OMIDVAR, LUTZ STRAẞBURGER, *Combinatorial Flows and their compositions.*
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The modern form of proof theory was established by David Hilbert with his project on the foundations of Mathematics, famous as Hilbert's problems posed in the 19's. The 24th problem of Hilbert [8], which asks about the possibility of comparing proofs, is as old as proof theory since it is the nature of the study. So far proof theorists have two completely different approaches to this problem. The first is to find suitable proof transformations and postulate that they are the same if they can be transformed into each other using these transformations. This can be achieved via proof normalization [7] or rule permutations [6]. However, transformations such as cut normalization affect proof complexity drastically. The second approach to the problem is to define suitable canonical proof representations. The most prominent examples are λ-terms [3], proof nets [2], and combinatorial proofs [4]. Our approach, by introducing combinatorial flows, classifies as the latter among the two approaches. We introduce combinatorial flows as a graphical representation of proofs in classical propositional logic with the possibility of showing the additive and multiplicative nature of the logic. Combinatorial proofs can be seen as a generalization of atomic flows [5] or combinatorial proofs. From atomic flows they inherit the close correspondence with open deduction and deep inference and the possibility of tracing the occurrences of atoms in a derivation. From combinatorial proofs, introduced by Hughes, they inherit the correctness criterion that allows the reconstruction of the derivation from the flow. In fact, combinatorial flows form a proof system in the sense of Cook and Reckhow [1]. We show how to translate between open deduction derivations and combinatorial flows, how to trace atoms in the proof, and how combinatorial flows are related to combinatorial proofs with cuts.

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[7] Dag Prawitz, ***Natural Deduction, A Proof-Theoretical Study***, Almquist and Wiskell, 1965.

[8] Rüdiger Thiele, *Hilbert's Twenty-Fourth Problem*, ***American Mathematical Monthly***, vol. 110 (2003), pp. 1–24.

▸ EUGENIO ORLANDELLI, AND MATTEO TESI, *A proof-theoretic approach to monadic logic.*

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Monadic logic is well-known to constitute a decidable fragment of first-order logic. The proof of the decidability result dates back to Löwenheim who employed model-theoretic methods [1]. A different proof of the same result was offered by Quine and published in the Journal of Symbolic Logic [3]. The key feature of Quine's proof is the use of a transformation of monadic formula in a certain normal form to which we refer as *innex* normal form. In essence, every monadic formula is equivalent to a formula which is a boolean combination of atomic formulas and existentially (universally) quantified finite conjunctions (disjunctions).

We propose a proof-theoretic approach to the issue. In particular, we first give a detailed reconstruction of Quine's procedure to reduce formulas in innex normal form via cuts and provable equivalences. Next, we introduce a sequent calculus for formulas in innex normal form which enjoys strong termination of the proof search in the sense that every bottom-up sequence of applications of the rules terminates leading to a derivation or to a finite countermodel.

A thorough structural analysis of the system is carried out by showing the admissibility of the structural rules of weakening, contraction and cut. The analysis is interesting as it is peculiar of the system due to certain context-restrictions imposed on the rules. Also, the cut-elimination strategy is obtained arguing by a single inductive parameter - the degree of the cut formula - rather than by induction on lexicographically ordered pairs - the degree of the cut formula and the sum of the height of the derivations of the premises of the cut - as usual in first-order languages (a proof running by induction on a single inductive parameter - different from the degree of the cut formula - for classical propositional logic can be found in [2]).

Finally, applications to the theory of syllogisms are described and themes for future research are briefly sketched.

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▸ LEONARDO PACHECO, *The μ-calculus' collapse on variations of* S5.
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The μ-calculus is obtained by adding to modal logic the least and greatest fixed-point operators μ and ν. The alternation depth of a formula measures the entanglement of its least and greatest fixed-point operators. Bradfield [2] showed that, for all $n \in \mathbb{N}$, there is a formula W_n such that W_n has alternation depth n and, over all Kripke frames, W_n is not equivalent to any formula with alternation depth smaller than n.

The same may not happen over restricted classes of frames: Alberucci and Facchini [1] showed that, over frames of S5, every μ-formula is equivalent to a formula without fixed point operators. In this case, we say the μ-calculus collapses to modal logic over frames of S5.

We show how Alberucci and Facchini's proof generalize to the μ-calculus's collapse over frames of intuitionistic S5. This generalization can also be done for some non-normal logics and for graded modal logics. We also show that, on the other hand, the μ-calculus does not collapse over the bimodal logic $\mathsf{S5}_2$.

[1] Luca Alberucci and Alessandro Facchini, *The modal μ-calculus hierarchy over restricted classes of transition systems*, ***The Journal of Symbolic Logic***, vol. 74 (2009), no. 4, pp. 1367-1400.

[2] Julian C. Bradfield, *The modal mu-calculus alternation hierarchy is strict*, ***Theoretical Computer Science***, vol. 195 (1998), no. 2, pp. 133–153.

▸ FEDOR PAKHOMOV, *How to escape Tennenbaum's theorem..*

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A celebrated result of Stanley Tennenbaum[1] states there are no computable non-standard models of first-order Peano arithmetic PA. In the present talk I will address the question of how sensitive this theorem is to the exact choice of signature. Namely I will present a construction of a theory definitionally equivalent to first-order Peano arithmetic PA that has a non-standard computable model. The same technique allows us to construct a theory definitionally equivalent to Zermelo- Fraenkel set theory ZF that has a computable model.

[1] Stanley Tennenbaum, *Non-archimedean models for arithmetic*, ***Notices of the American Mathematical Society***, 6(270):44, 1959.

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▶ LUIZ CARLOS PEREIRA, ELAINE PIMENTEL, AND VALERIA DE PAIVA, *Translations and Prawitz ecumenical system.*
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Ecumenical systems are formal codifications where two or more logics can co-exist in peace, which means that these logics accept and reject the same formulae, the same rules and the same basic principles. Dag Prawitz proposed a natural deduction ecumenical system [2], where classical logic and intuitionistic logic are codified in the same system (see also [1]). In this system, the classical logician and the intuitionistic logician would share the universal quantifier, conjunction, negation and the constant for the absurd, but they would each have their own existential quantifier, disjunction and implication, with different meanings. Prawitz main idea is that these different meanings are given by a semantic framework that can be accepted by both parties. The rules for the intuitionistic operators ($\rightarrow_i, \vee_i, \exists_i$) and for the shared operators ($\wedge, \neg, \bot, \forall$) are the usual Gentzen-Prawitz natural deduction introduction and elimination rules. The rules for the classical propositional operators are as follows:

$$\frac{\begin{array}{c}[A] \quad [\neg B] \\ \Pi \\ \bot\end{array}}{(A \rightarrow_c B)} \rightarrow_c\text{-int} \qquad \frac{(A \rightarrow_c B) \quad A \quad \neg B}{\bot}$$

$$\frac{\begin{array}{c}[\neg A] \quad [\neg B] \\ \Pi \\ \bot\end{array}}{(A \rightarrow_c B)} \vee_c\text{-int} \qquad \frac{(A \vee_c B) \quad \neg A \quad \neg B}{\bot}$$

This short note has two main objectives. The first is to show, in the propositional case, that there are interesting relations between the Gödel-Gentzen translation and the ecumenical perspective, but that the later cannot be reduced to the former. The second main objective is to investigate the possibility of ecumenical systems with two independent negations, one classical and one intuitionistic.

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[2] Prawitz, D., *Classical versus intuitionistic logic*, ***Why is this a Proof?, Festschrift for Luiz Carlos Pereira*** (Hermann Haeusler, Wagner Sanz, and Bruno Lopes editors), College Books, UK, 2015, pp. 15–32.

▸ IOSIF PETRAKIS, DANIEL WESSEL, *Swap algebras and swap rings.*
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Following [1], an equality and an inequality on a set X induce the positive notions of disjoint subsets and of complemented subsets of X. Complemented subsets are easier to handle than plain subsets, as their partial, characteristic functions are constructively defined and their complement, formed by swapping its components, behaves like the classical complement of a subset. Complemented subsets are crucial to the constructive reconstruction of the classical Daniell approach to measure theory. We explain why the pair of notions (complemented subsets, boolean-valued partial functions) is the constructive analogue to the classical pair (subsets, boolean-valued total functions). Following [2, 3], we introduce swap algebras of type (I) and (II) as an abstract version of Bishop's algebras of complemented subsets of type (I) and (II), respectively, and swap rings as an abstract version of the boolean-valued partial functions on a set. We present several results indicating that the theory of swap algebras and swap rings is a generalisation of the theory of boolean algebras and boolean rings.

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▸ PERCEVAL PILLON, *Deontic logic as a formalisation puzzle*.

Deontic logic as a formalisation puzzle Deontic logic is a branch of formal logic aiming to give a formal account of a set of notions revolving notably around obligation and normative reasoning. It has been studied by a large range of formal systems of different natures [1] and grew to address more and more philosophical questions. Our aim is to study it from the perspective of a critical theory of formalization given the specificity and the social anchoring of the notions at stake. Philosophical goals for the formalization of the set of notions in play will be identified and questioned and we will study the adequation between them and the formal tool used to give logical systems. By formal tools we mean here the choice of formal solutions chosen in order to formalize the aimed notion. The main point can be summarized by the question "why and how one formalizes such notions ?". Our proposal will rely on two main philosophical tools to carry out the analysis: the Carnapian notion of formal explication and the literature on the theory of formalization (works by C. Dutilh Novaes for example). Given the large scope of the question, we will focus on significant examples of formal propositions : systems of modal deontic logic and in and out logic. Our communication will first briefly explain the historical development of the field of deontic logic in order to contextualize the examples. Then we will present the criteria used to analyze the formal. The last and main part of the talk will analyze the selected systems, by applying the criteria defined earlier to them, having in mind the philosophical or practical goals they are supposed to achieve.

[1] Hilpinen, R., and McNamarra, P. (2013). Deontic Logic a Historical survey and Introduction.

▶ PEDRO PINTO, *Proof mining and the convex feasibility problem.*
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In this talk, we will discuss a recent proof mining study [1] regarding the strong convergence of Dykstra's algorithm. Proof mining [2] employs proof-theoretical techniques to analyse *prima facie* noneffective mathematical proofs with the goal of extracting additional information. Such new information is usually in the form of effective and highly uniform rates or bounds. In the last twenty-five years, this area of Proof Theory has been greatly developed by Ulrich Kohlenbach and his collaborators, and proof mining techniques have been particularly successful in applications to nonlinear analysis and adjacent areas. In convex optimization, many practical problems can be framed in the setting of the convex feasibility problem [3], i.e. finding the projection onto the intersection of finitely many convex sets under the assumption that the projection onto the individual sets is easy to compute. We will discuss new results regarding the asymptotic behavior of the well-known Dykstra's algorithm [4, 5], obtained via proof mining techniques.

[1] Pedro Pinto, *On the finitary content of Dykstra's algorithm*, ***Manuscript in preparation***, (2023).

[2] Ulrich Kohlenbach, ***Applied proof theory: proof interpretations and their use in mathematics***, Springer Monographs in Mathematics, Springer, 2008.

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[5] James P. Boyle and Richard L. Dykstra, *A Method for Finding Projections onto the Intersection of Convex Sets in Hilbert Spaces*, ***Advances in Order Restricted Statistical Inference. Lecture Notes in Statistics, vol. 37*** (Richard L. Dykstra, Tim Robertson and Farroll T. Wright, editors), Springer, New York, 1986, pp. 28–47.

▸ PHILIPP PROVENZANO, *Extracting ω-models from well-ordering principles.*
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It is known by works of Girard that arithmetical comprehension is equivalent to the following well-ordering principle: $\forall\alpha.\,\mathrm{WO}(\alpha) \to \mathrm{WO}(\omega^\alpha)$, where $\omega^\bullet$ denotes an exponentiation operation on linear orders. Iterating this principle establishes a connection between preservation of well-orders by the Veblen function φ, which can be viewed as transfinitely iterated exponentiation, to corresponding comprehension schemes $\Pi^0_\alpha - \mathrm{CA}_0$, denoting α-iterated Π^0_1 comprehension. The latter theory is more commonly known as ATR_0 and it is a famous result by Friedman that it is equivalent to the well-ordering principle $\forall\alpha.\,\mathrm{WO}(\alpha) \to \mathrm{WO}(\varphi_\alpha(0))$. Relativizations of this iteration principle, starting from arbitrary sufficiently well-behaved transformations of well-orders, have been studied in my master thesis, using ordinal analysis as a proof-theoretic tool. The goal of this talk is to give a more direct computability theoretic account of the result.
The comprehension principles $\Pi^0_\alpha - \mathrm{CA}_0$ can be more conveniently stated as iterated existence of ω-models. These are models of arithmetic with standard first-order domain and countable set-universe. The principle $\omega\,\mathrm{Con}(T)$ states that ω-models of a theory T exist relative to any given set. Iterating this leads to the principles

$$\omega\,\mathrm{Con}^\alpha(T) := \omega\,\mathrm{Con}\left(T + \left\{\omega\,\mathrm{Con}^\beta(T) \,\middle|\, \beta \prec \alpha\right\}\right)$$

and we get $\Pi^0_{\omega^{1+\alpha}} - \mathrm{CA}_0 \equiv \omega\,\mathrm{Con}^\alpha(\mathrm{ACA}_0)$. For a sufficiently well-behaved transformation D of well-orders, our main result can then be stated as the equivalence between the principle of $D^\bullet$ preserving well-orders and

$$\forall\alpha.\,\mathrm{WO}(\alpha) \to \omega\,\mathrm{Con}^\alpha(\mathrm{ACA}_0 +\text{"}D \text{ preserves well-orders"}),$$

where $D^\bullet$ denotes a suitably defined transfinite iteration of D. In order to give a direct proof of this result, we want to reduce the task of finding a universe for such ω-models to the computational problem of finding an infinite path in α when given an infinite path in $D^\alpha(0)$ for a linear order α. The latter is just the computational content of the statement that $D^\bullet(0)$ preserves well-orders.
For the case of $D = \omega^\bullet$, this has been answered by Marcone and Montalbán in [1]. I show how their method relativizes to arbitrary transformations of well-orders by associating a corresponding set-existence operator.

[1] ALBERTO MARCONE, ANTONIO MONTALBÁN, *The Veblen functions for computability theorists*, ***The Journal of Symbolic Logic***, vol. 76 (2011), no. 2, pp. 575–602.

▸ JONI PULJUJÄRVI, DAVIDE EMILIO QUADRELLARO, *Some Model-Theoretic Results in Team Semantics*.
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In this talk we continue the work started in [1] and we try to develop a suitable model-theoretic framework for logics over team semantics. In fact, since logics in team semantics admit a compactness theorem [1], it is natural to consider how far the standard tools and results from classical model theory can be pushed in this context.

We introduce a suitable notion of maps between model that preserve formulas of independence logic and we describe the resulting category of models and morphisms. In particular, we show that a suitable version of the amalgamation property holds in this context and we introduce a notion of Galois types for logics in team semantics. Finally, we also describe in better details to what extent this category fits the framework of Abstract Elementary Categories of Kamsma and Kirby [2].

[1] Joni Puljujärvi and Davide Emilio Quadrellaro, *Compactness in Team Semantics*, https://arxiv.org/abs/2212.03677.

[2] Mark Kamsma, *The Kim-Pillay Theorem for Abstract Elementary Categories*, The Journal of Symbolic Logic 85, no. 4 (2020): 1717–41.

▸ MARCOS M. RECIO, JOSÉ MIGUEL BLANCO, SANDRA M. LÓPEZ, *Exploring the building blocks of 2 set-up Routley-Meyer semantics.*
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While 2 set-up Routley-Meyer semantics has been historically considered as less interesting than regular Routley-Meyer semantics, this trend seems to break in recent years [1]. In particular, its accessibility relation has been studied with respect to the semantic postulates of its regular counterpart in certain many-valued logics [2]. Nevertheless, there is another relationship to be explored within 2 set-up Routley-Meyer semantics: the one of the accessibility relation with respect to Hilbert-style theorems. For each of the eight possible accessibility relations supported by 2 set-up Routley-Meyer semantics, there is an intrinsic relationship with, at least, one theorem that needs to be part of the logical system endowed with this kind of semantics. Thus, the main aim of this talk is to present an approach to how the accessibility relations relate to Hilbert-style theorems and how each other supports themselves during the process of soundness and completeness proofs.

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[2] J. M. Blanco, S. M. López, M. M. Recio, *On how the 2 Set-up Routley-Meyer semantics are a case of the Reduced General Routley-Meyer semantics in the context of some 4-valued logics*, ***Journal of Applied Logics***, vol. 10 (2023), no. 1, pp. 1–18.

▶ SAM SANDERS, *The Biggest Five of Reverse Mathematics*.
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I provide an overview of joint work with Dag Normann on the higher-order Reverse Mathematics (RM for short) of the Big Five systems and the surprising limits of this enterprise ([3]).

The well-known *Big Five phenomenon* of RM is the observation that a large number of theorems from ordinary mathematics are either provable in the base theory or equivalent to one of only four systems; these five systems together are called the 'Big Five' of RM. The aim of this paper is to **greatly** extend the Big Five phenomenon, working in Kohlenbach's *higher-order* RM ([1]).

In particular, we have established numerous equivalences involving the **second-order** Big Five systems on one hand, and well-known **third-order** theorems from analysis about (possibly) discontinuous functions on the other hand. We both study relatively tame notions, like cadlag or Baire 1, and potentially wild ones, like quasi-continuity. We also show that *slight* generalisations and variations (involving e.g. the notions Baire 2 and cliquishness) of the aforementioned third-order theorems fall *far* outside of the Big Five. In particular, these slight generalisations and variations imply the principle $\mathsf{NIN}_{[0,1]}$ from [2], i.e. there is no injection from $[0,1]$ to $\mathbb{N}$. We have no explanation for this phenomenon.

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[2] Dag Normann and Sam Sanders, *On the uncountability of* $\mathbb{R}$, Journal of Symbolic Logic, DOI: `doi.org/10.1017/jsl.2022.27` (2022), pp. 43.

[3] ———, *The Biggest Five of Reverse Mathematics*, Submitted, arxiv: `https://arxiv.org/abs/2212.00489` (2023), pp. 39.

▶ LUCA SAN MAURO, *Effective analogues of the countable Borel equivalence relations.*
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Coskey, Hamkins and Miller [1] proposed two possible analogues of the class of countable Borel equivalence relations in the setting of computable reducibility of equivalence relations on the computably enumerable (c.e.) sets. The first is based on effectivizing the Lusin-Novikov theorem while the latter is based on effectivizing the Feldman-Moore theorem. They ask for an analysis of which degrees under computable reducibility are attained under each of these notions.

We investigate these two notions, in particular showing that the latter notion has a strict dichotomy theorem: *Every such equivalence relation is either equivalent to the relation of equality ($=^{ce}$) or almost equality (E_0^{ce}) between c.e. sets.* For the former notion, we show that this is not true, but rather there are both chains and antichains of equivalence relations on c.e. sets which are enumerable in the indices and between E^{ce} and E_0^{ce}. This gives several strong answers to [1, Question 3.5] showing that in general there is no analogue of the Glimm-Efros dichotomy for equivalence relations on the c.e. sets.

This is joint work with Uri Andrews.

[1] S. Coskey, J.D. Hamkins, and R. Miller, *The hierarchy of equivalence relations on the natural numbers under computable reducibility*, forthcoming in ***Computability*** 1(1):15–38, 2016

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Can logic change over time? On the one hand, the logical concepts, as expressed by function words (*every, some, and, if*), are subject to the evolutionary forces shaping natural language vocabulary. Since natural language undergoes constant and continuous change, so do the logical concepts expressed through it. On the other hand, the logical operators are unchanging: as part of the abstract mathematical realm there can be no more change in logic than there can be in mathematics.

Our goal is to make some headway on a possible reply to this dilemma. We begin by characterizing two senses of the word 'logic', distnguishing, following Harman [2], between a *theory of deduction* and a *theory of reasoning*. This distinction is used to defuse Quine's [6] famous objection to the possibility of change in logic: according to Quine, there can only be wholesale replacement of logical theory but no incremental development ('change of logic, change of subject'). We then present two arguments in favor of the possibility of change in logic, one from a naturalistic perspective on scientific explanation [3] and the other from considerations of open texture [7, 4].

Having argued for change in logic, we owe an account of logical meaning that, on the one hand, shows how logical concepts can change while, on the other, explains their relative robustness when it comes to conceptual change: the logical vocabulary can change, but not as fast as nouns and predicates do. To this end we first discuss a proposal based on Došen's [1] idea that the logical constants mark structural features of deductive reasoning. We then reject the problematic underlying assumption of a stable *core meaning*, to sketch an account that makes room for a more flexible treatment of the identity and individuation of logical concepts, elaborating on an old theme from Putnam [5].

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▸ ANDREA SERENI, MARIA PAOLA SFORZA FOGLIANI, LUCA ZANETTI,
Pluralism about Criteria.
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Logic has historically been thought to enjoy a series of properties that make its status to various degrees privileged – in particular, its propositions have long been regarded to be *a priori*, unrevisable, necessary and analytic; moreover, the set of such laws has generally been taken to be unique. These traditional conceptions on the nature of logic have further often come coupled with a particular system, *viz.* classical logic (CL). However, more or less recently, some serious stabs have been taken at dismantling all of these assumptions. Several counterexamples even to the most basic laws and rules of inference of classical logic have been advanced to begin with, the most notable example of which is arguably [3]. Moreover, logical pluralists (for example [1]) took the task of defeating logic's uniqueness, by arguing that we should be accepting that there is in fact more than one correct logic. Finally, anti-exceptionalists about logic (for example [5], [4], [2]) have argued, building on Quinean ideas, that logic does not deserve any special place among the other disciplines for its laws do not possess any of the above mentioned features, and that its theories can be justified, revised, and compared just as scientific theories are – that is, by means of a broadly abductive methodology.

It is a merit of the recent discussion on anti-exceptionalism to have brought a focus on selection *criteria* that underlie theory-choice in logic. These criteria differ both in their *kind* and in the *number* of logics they select. In the exceptionalism *vs.* anti-exceptionalism debate, for example, traditional criteria such as analyticity, aprioricity, formality, necessity, etc. are opposed to abductive criteria such as simplicity, adequacy to the data, consistency, strength, etc. Orthogonal to this dispute, the logical monism *vs.* logical pluralism debate sees positions that contend that the relevant criteria can select at most one logic opposing positions that claim that more than logic is selected by those criteria and therefore legitimate. This tends to generate a wide combinatorics of positions: on the two ends, classical exceptionalist monists and deviant anti-exceptionalist pluralists; in between, a spectrum of anti-exceptionalist monists and exceptionalist pluralists, each with their logic or logics of choice.

Unfortunately, this can also quickly render disputes at cross-purposes, as theorists soon start talking past each others. So, for instance, classical monists might argue that CL has an edge in virtue of its simplicity, and/or widespread use in arithmetical, and hence scientific, practice; deviant logicians will not put this into question, but answer that – say – CL is unable to account both for our intuitions on natural language implications, and for longstanding semantic and set-theoretic paradoxes. That is, disagreement appears to rest not only (and, arguably, not prominently) on the various theories' own merits – *e.g.* on whether the material conditional really captures our informal use of "if ... then" statements – but rather on the grounds or criteria on the bases of which we should evaluate them – *e.g.* on whether natural language intuitions have a say in our selection of a logical theory. Similarly, pluralists will not object to the advantages of classical (or any other) logic in particular fields or applications, but rather claim that the grounds for selection compel us to sanction other, equally legitimate, theories.

An arguably more fruitful and cogent strategy would consist in going the other way around – by first specifying a set of criteria for establishing when a theory is to be considered a legitimate logic, and only then by evaluating which candidate or candidates satisfies or satisfy those criteria. To these criteria we now turn. First, we survey a number of possible criteria for theory-choice, divided into different categories according to whether they count as exceptionalist or anti-exceptionalist, on the one hand, and whether they can be used to support logical monism or logical pluralism, on the other. We then suggest a new classification of possible views in the philosophy of logic based on the kind (exceptionalist or anti-exceptionalist) and number (monist or pluralist) of the criteria they adopt. We show how such novel classification suggests a form of pluralism about criteria themselves and we compare it with exceptionalism and anti-exceptionalism about logic. Finally, we offer some concluding remarks and arguments in support of pluralism about criteria.

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▸ ILYA SHAPIROVSKY, *Locally finite polymodal logics and Segerberg – Maksimova criterion.*
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A logic L is locally finite (in another terminology, *locally tabular*), if each of its finite-variable fragments contains only a finite number of pairwise nonequivalent formulas. In algebraic terms, it means that the variety of L-algebras is locally finite, that is every finitely generated L-algebra is finite.

An important characterization of local finiteness was obtained by K. Segerberg [2] and L. Maksimova [1] in 1970s for unimodal transitive logics: in this case, a logic is locally finite if and only if it contains a modal formula B_n of finite height. This is a nice theorem in many respects. Besides the fact that it gives a natural semantic criterion of local finiteness, it also provides an axiomatic characterization: local finiteness is expressed by a formula from an explicitly described set $\{B_n \mid n < \omega\}$. No axiomatic criterion of local finiteness is known for polymodal, and even for all (including non-transitive) unimodal logics. In [4], Segerberg – Maksimova criterion was extended for some families of unimodal non-transitive logics.

In this talk I will discuss generalizations of this criterion in the polymodal context. Some of the results are based on the recent work [3].

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▸ ANDREI SIPOŞ, *The computational content of super strongly nonexpansive mappings*.
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Strongly nonexpansive mappings are a core concept in convex optimization. Recently, they have begun to be studied from a quantitative viewpoint: U. Kohlenbach has identified in [2] the notion of a 'modulus' of strong nonexpansiveness, which leads to computational interpretations of the main results involving this class of mappings (e.g. rates of convergence, rates of metastability). This forms part of the greater research program of 'proof mining', initiated by G. Kreisel and highly developed by U. Kohlenbach and his collaborators, which aims to apply proof-theoretic tools to extract computational content (which may not be immediately apparent) from ordinary proofs in mainstream mathematics (for more information on the current state of proof mining, see the book [1] and the recent survey [3]). The quantitative study of strongly nonexpansive mappings has later led to finding rates of asymptotic regularity for the problem of 'inconsistent feasibility' [4, 7], where one essential ingredient has been a computational counterpart of the concept of rectangularity, recently identified in [5] as a 'modulus of uniform rectangularity'.

Last year, Liu, Moursi and Vanderwerff [6] have introduced the class of 'super strongly nonexpansive mappings', and have shown that this class is tightly linked to that of uniformly monotone operators. What we do is to provide a modulus of super strong nonexpansiveness, give examples of it in the cases e.g. averaged mappings and contractions for large distances and connect it to the modulus of uniform monotonicity. In the case where the modulus is supercoercive, we give a refined analysis, identifying a second modulus for supercoercivity, specifying the necessary computational connections and generalizing quantitative inconsistent feasibility.

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▶ GIOVANNI SOLDÀ, *On the reverse mathematics of some results of bqo theory.*
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Well quasi-orders (or wqo's) are a very natural kind of structure, which arises in many different areas of mathematics. Unfortunately, they do not enjoy very strong closure properties, which makes it somewhat difficult to work with them. This sort of consideration is what led Nash-Williams to define a stronger notion, namely, that of better quasi-order (or bqo's), which was to prove central in the proof of several results in order theory, chiefly among them Fraïssé's Conjecture. The study of these results from the perspective of reverse mathematics is an important and active line of research, a survey of which can be found in [1].

In this talk, we will revisit a classical stratification of the concept of bqo-ness for a quasi-order P, namely α-bqo-ness, and study the relationship it has with the set of iterated downward-closed sets of P and the class of the indecomposable sequences with values in P, over the theory ATR_0. Finally, we will use this to prove, in ATR_0, the classical result that P is bqo if and only if the set of sequences with values in P is wqo (a result known to imply ATR_0 over RCA_0).

The results presented are joint work with Fedor Pakhomov.

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▶ IOANNIS SOULDATOS, *The Hanf Number for the Joint Embedding Property*.
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Define the Hanf number for the joint embedding property (JEP), or the amalgamation property (AP), for Abstract Elementary Classes (AEC) to be the least cardinal μ so that if $\boldsymbol{K}$ is an AEC with $LS(\boldsymbol{K}) < \mu$,and $\boldsymbol{K}$ satisfies JEP (AP) cofinally below μ, then $\boldsymbol{K}$ satisfies JEP (AP) in all cardinals $\geq \mu$.

In [1], Baldwin and Boney proved that the first strongly compact cardinal is an upper bound for the Hanf number for JEP and AP. They raised the question if the strongly compact upper bound is optimal.

In this talk we will survey some recent developments in the area.

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▸ BERNHARD STOINSKI, *Extension of category theory using a PL0 calculus functor to form propositional morphisms in multi-agent systems.*

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Abstract: The subject of this talk are morphisms between categories that are able to map truth values by means of a PL0-calculus functor $Calc_0$. This functor is used in the generation of AI multi-agent systems (MAS) [3]. In this case, the agents are equivalent to the categories [1]. A highly simplified example of an AI-MAS using PL0 calculus functors represents the practical aspect of this talk [2]. The special feature of the functor $Calc_0 : A \to B$ is that the morphism from agent A to agent B yields a truth value $t_A : X \to [0, 1]$, taking A to be a fuzzy set. The function value $m_A(a)$ for $a \in X$ is itself again the membership value formed by the result of a calculus function of A. Hereby A itself becomes a fuzzy set. By this fact, a fuzzy space is formed by means of $Calc_0$, which, however, must not be confused with a type 2 fuzzy set. Through this construct and the self-similarity of the MAS, it is possible to represent complex natural processes with a high entropy [4] content.

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▸ JUAN M SANTIAGO SUÁREZ, MATTEO VIALE, *Boolean valued semantics for infinitary logics.*

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It is well known that the completeness theorem for $L_{\omega_1\omega}$ fails with respect to Tarski semantics. Mansfield [1] showed that it holds for $L_{\infty\infty}$ if one replaces Tarski semantics with boolean valued semantics. We use forcing to improve his result in order to obtain a stronger form of boolean completeness (but only for $L_{\infty\omega}$). Leveraging on our completeness result, we establish the Craig interpolation property and a strong version of the omitting types theorem for $L_{\infty\omega}$ with respect to boolean valued semantics. We also show that a weak version of these results holds for $L_{\infty\infty}$ (if one leverages instead on Mansfield's completeness theorem). Furthermore we bring to light (or in some cases just revive) several connections between the infinitary logic $L_{\infty\omega}$ and the forcing method in set theory.

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► DAVIDE SUTTO, *Potentialist set theory: New paths and open questions.*
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In the last ten years potentialist set theory has emerged as one of the most lively trends in the philosophy of set theory. Remarkably, a modal account of sets has been developed in two different ways, the first inspired by the work of Charles Parsons and the second dating back to Hilary Putnam and Geoffrey Hellman. The aim of the paper is to present these two approaches through two groups of questions, with the aim of outlining the state of the art while, at the same time, sketching the new paths and challenges soon-to-be-faced by a potentialist account of sets.

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▸ AMIRHOSSEIN AKBAR TABATABAI, AND RAHELEH JALALI, *Feasible admissible rules in intuitionistic modal logics*.

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In this talk, we introduce a general family of sequent-style calculi over the modal language and its fragments to capture the essence of all constructively acceptable systems. Calling these calculi *constructive*, we show that any strong enough constructive sequent calculus, satisfying a mild technical condition, feasibly admits all Visser's rules, i.e., there is a polynomial time algorithm that reads a proof of the premise of a Visser's rule and provides a proof for its conclusion. As a positive application, we show the feasible admissibility of Visser's rules in several sequent calculi for intuitionistic modal logics, including CK, IK and their extensions by the modal axioms T, B, 4, 5, the modal axioms of bounded width and depth and the propositional lax logic. On the negative side, we show that if a strong enough intuitionistic modal logic (satisfying a mild technical condition) does not admit at least one of Visser's rules, then it cannot have a constructive sequent calculus. Consequently, no intermediate logic other than IPC has a constructive sequent calculus.

▸ AMIRHOSSEIN AKBAR TABATABAI, *Mining the Surface: Witnessing the Low Complexity Theorems of Arithmetic*.
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One of the elegant achievements in the history of proof theory is the characterization of the provably total recursive functions of a theory by its proof-theoretic ordinal as a measure for the time complexity of the functions. Unfortunately, this characterization is not sufficiently fine-grained to capture the subclass of the functions with a feasible (polynomial time) definition. In this talk, we fill this gap. We show that if α is the proof-theoretic ordinal of the theory T and it has a polynomial time representation, then a feasibly-defined function is provably total in T iff it is computable by a sequence of PV-provable polynomial time modifications on a PV-provable initial polynomial time value, where the computational steps are indexed by the ordinals below α, decreasing by the modifications.

▸ HAKOB TAMAZYAN, *Comparison of proof complexities for linear proofs in quantified sequent calculus and substitution sequent calculus.*
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A. Carbone proved in [1] that there is an exponential speed-up in the number of lines of the quantified propositional sequent calculus (QPK) [2] over substitution sequent calculus (SPK) [2] for tree-like proofs. This paper investigates the transformation of linear QPK-proof of any quantifier-free tautology into a linear SPK-proof of the same tautology. An algorithm is proposed for such transformation, which is then used to prove the following two statements.

Theorem 1. For a given linear proof of some quantifier-free tautology in QPK with t number of lines, exists some linear proof of the same tautology in SPK, having $O(t^2)$ number of lines.

Theorem 2. For a given linear proof of some quantifier-free tautology in QPK with proof size s, exists some linear proof of the same tautology in SPK, having $O(s^5)$ proof size.

Since SPK is polynomially equivalent to the substitution Frege systems (SF) [2], there is a transformation of a linear proof of any quantifier-free tautology in QPK into a linear proof of the same tautology in SF with no more than polynomially increases of the proof lines and size. The obtained results show that the QPK system doesn't have a substantial advantage over the systems SPK and SF in the terms of linear proofs.

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I argue that two main issues arise in connection with the justification of Large Cardinal Axioms: one is the 'Intrinsicness Issue', namely, the fact they do not seem to straightforwardly follow from the 'concept of set' (the 'iterative concept of set'). The second is what I call the 'Universality Issue', the fact that there does not seem to exist any intrinsically motivated principle (or class of principles) equivalent to all known Large Cardinals. My strategy to tackle both issues is as follows. First, I examine the main 'abstract motivating principles' (in particular, Reflection, Resemblance and Uniformity) which have been introduced (and invoked) to justify large cardinals. As has been pointed out, such principles are not licensed by the concept of set only, insofar as they entail (or are best construed as entailing) the existence of classes. As a consequence, in the paper, I discuss several strategies to 'stretch' the concept of set so as to legitimise the use of (at least, some) class theory. Then, I proceed to review Bagaria's Structural Reflection Principles which may turn out to be optimal in terms of justificatory strength, since they are motivated by several abstract principles (in particular, Resemblance and Uniformity), entail the use of a modest amount of class theory and may also be able to account for the existence of practically all known large cardinals.

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Non-epistemic criteria, including pragmatic considerations, are not indispensable when choosing between rival logics; rational or epistemic adjudication is possible. This is a position that has recently been defended by Ian Rumfitt [1]. So he argues, more particularly, that classical logicians have no serious epistemic reason for dropping the distribution of conjunction over disjunction, i.e., for adopting quantum logic. I disagree with Rumfitt, and the paper explains why.

I give a formal version of the proof (henceforth, "the Proof") that distribution is false in standard quantum mechanics, making use of the rules for $\wedge$-introduction and $\wedge$-elimination, substitution and De Morgan only, all available in quantum logic. Then, I describe Rumfitt's first truth-ground semantics ($\mathcal{TG}_1$) and his criticism of the Proof: $\mathcal{TG}_1$ cannot validate the behavior of disjunction in the Proof, so a metalogical proof is needed to justify that behavior. But this proof requires classical logic, which makes the Proof rule-circular. Afterwards, I present Rumfitt's second truth-ground semantics ($\mathcal{TG}_2$) including an adjusted semantic principle for disjunction, and then his criticism: the Proof assumes that the state space of a physical system is finite-dimensional, so it must be revised. But Rumfitt argues that the revised Proof is unsound with respect to $\mathcal{TG}_2$. This is supported by the so-called Eigenstate-Eigenvalue Link (EEL), which implies that observables with a continuum spectrum, like position and momentum, have no precise values because they have no eigenstates if represented on an infinite-dimensional space. Rumfitt concludes that the classical logician doesn't have any good epistemic reason to believe that distribution can be dropped.

However, I think this conclusion is not justified. I first argue that despite dropping the assumption of finite-dimensionality, Rumfitt has not shown that the revised Proof is unsound with respect to $\mathcal{TG}_2$. For although one cannot simply reject the EEL, to allow observables to have precise values at all times, rather than only at eigenstates, one can nevertheless revise the EEL as well, by coarse-graining the infinite-dimensional state space. This allows observables with a continuous spectrum to have coarse-grained values at regions, rather than at points in that space. The revised Proof is sound with respect to a coarse-grained $\mathcal{TG}_2$. Secondly, I reconsider $\mathcal{TG}_1$ in order to argue that Rumfitt has not shown the Proof rule-circular, either. I explain rule-circularity away by showing why quantum connectives can behave classically in the metalanguage of the Proof. To explain this classical recapture, more coarse-graining is needed, of a kind that is unavoidable in quantum mechanics (since implied by no-go theorems like Kochen-Specker). Quantum connectives can behave classically not only in the metalanguage of quantum mechanics, but in all contexts where the Uncertainty Principle does not apply.

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▸ AGATA TOMCZYK, *Sequent Calculus for non-Fregean topological Boolean theory* WT.
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The aim of the talk is to present sequent calculus $\mathsf{G3_{WT}}$ for WT, one of axiomatic extensions of non-Fregean logic SCI. This talk concerns recent work regarding proof theory for non-Fregean logis; no proof systems for WT has been examined thus far. WT is inspired by the following proposition from Wittgenstein's *Tractatus*:

> 5.141 If p follows from q and q from p then they are one and the same proposition.

which can be interpreted as follows: two logically equivalent sentences constitute different variants of the same proposition. In WT identity connective '$\equiv$' is weakened. We add more valid equations than it was in the case of SCI; a given equation $\phi \equiv \chi$ is valid in WT provided $\phi \leftrightarrow \chi$ is valid in SCI. Moreover, $\phi \equiv \chi$ can be translated to modal logic $\mathsf{S4}$ as $\Box(\phi \leftrightarrow \chi)$, where necessity operator $\Box$ can be interpreted as an interior operator on the Boolean algebra of situations. We will examine $\mathsf{G3_{WT}}$, which is based on the left-sided sequent calculus $\ell\mathsf{G3_{SCI}}$ proposed in [1]. In $\mathsf{G3_{WT}}$ we combine Negri's strategy of turning axioms into sequent calculus rules along with strategy of translating consequence operation properties into sequent calculus rules. We will discuss completeness and soundness of $\mathsf{G3_{WT}}$ with respect to topological Boolean algebra as well as point to limitations (and sources) regarding cut elimination procedure.

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▸ HSING-CHIEN TSAI, ZE-YUAN DUAN, *On the Complexity of First-order Axiomatizable Mereological Theories*.
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In this talk, we are only concerned with first-order mereological theories which can be axiomatized by using the following list of axioms that can be found in the literature.

(P1: reflexivity) $\forall x Pxx$
(P2: anti-symmetry) $\forall x \forall y ((Pxy \wedge Pyx) \rightarrow x = y)$
(P3: transitivity) $\forall x \forall y \forall z ((Pxy \wedge Pyz) \rightarrow Pxz)$
(EP: extensionality) $\forall x \forall y (\exists z PPzx \rightarrow (\forall z (PPzx \leftrightarrow PPzy) \rightarrow x = y))$
(WSP: weak supplementation) $\forall x \forall y (PPxy \rightarrow \exists z (PPzy \wedge \neg Ozx))$
(SSP: strong supplementation) $\forall x \forall y (\neg Pyx \rightarrow \exists z (Pzy \wedge \neg Ozx))$
(FS: finite sum) $\forall x \forall y (Uxy \rightarrow (\exists z \forall w (Owz \leftrightarrow (Owx \vee Owy)))$
(FP: finite product) $\forall x \forall y (Oxy \rightarrow \exists z \forall w (Pwz \leftrightarrow (Pwx \wedge Pwy)))$
(A: atomicity) $\forall x \exists y (Pyx \wedge \neg \exists z PPzy)$, where y is an "atom", for it has no proper part.
(AL: atomlessness) $\forall x \exists y PPyx$
(G: existence of the greatest member) $\exists x \forall y Pyx$
(C: complementation) $\forall x (\neg \forall z Pzx \rightarrow \exists z \forall w (Pwz \leftrightarrow \neg Owx))$
(UF: unrestricted fusion axiom schema) $\exists x \alpha \rightarrow \exists z \forall y (Oyz \leftrightarrow \exists x (\alpha \wedge Oyx))$, for any formula α where z and y do not occur free.

Previously, we have shown the following facts, where **CEM** is the theory axiomatized by (P1), (P2), (P3), (SSP), (FS) and (FP).

(1) Any theory strictly weaker than **CEM**+(C) is finitely inseparable and hence undecidable. (2) Any theory weaker than **CEM**+(G) is undecidable, but **CEM**+(G) is not finitely inseparable. (3) Any theory stronger than **CEM**+(C) is decidable.

In this talk, we will see that the undecidable theories mentioned in (1) or (2) are 1-complete and the decidable theories mentioned in (3) are NP-hard. Moreover, we did not deal with the atomless theories previously. Now we will show that any theory weaker than **CEM**+(G)+(AL) is 1-complete (note that an atomless theory cannot be finitely inseparable).

▸ IRIS VAN DER GIESSEN, RAHELEH JALALI, AND ROMAN KUZNETS, *Proving uniform interpolation via multicomponent sequent calculi*.

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Interpolation is one of the basic logical properties. Craig interpolation states that if $A \to B$ is valid, then there is an interpolant C with shared variables from A and B such that $A \to C$ and $C \to B$ hold. Uniform interpolation is stronger where the interpolant only depends on either A or B.

Proof-theoretic approaches to interpolation are key methods to construct interpolants. Cut-free sequent calculi are used for Craig interpolation and terminating sequent systems for uniform interpolation, i.e., [3]. For Craig interpolation, the proof-theoretic approach was extended to multicomponent sequents, such as hypersequents [4].

We provide new proofs of uniform interpolation in modal logic (see overview [1]), based on multicomponent sequents. The interpolants are defined proof-theoretically, but we use semantic bisimulation quantifiers to prove their correctness. We use terminating nested sequent calculi for logics K, T and D [2]. Recently, we introduced layered sequents for $K5, KD5, K45, KD45, KB45$ and $S5$ (coinciding with hypersequents for $S5$) and provide the first proof-theoretic proof of uniform interpolation for these logics.

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▸ GIORGIO VENTURI, PEDRO YAGO,

How to be (semantically) insensitive.

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In this paper we study the semantic insensitivity of nonnormal modal operators, in the sense of insensitivity presented in [5]. Starting with relational frames, we present an heuristic inspired by modalities that are insensitive to reflexivity [3], seriality [2] and narcissism [4], which we use to offer nonnormal modal languages which are insensitive to transitivity and symmetry. In the last part of the paper, we cover neighborhood models, presenting the insensitivity of particular nonnormal modal operators in the literature [1]. We then offer a general insensitivity result for a wide class of logics, which explain the previously presented particular insensitivities.

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▸ CHENG-SYUAN WAN, *Towards Skew Non-Commutative* MILL *with Additives.*
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This work concerns the proof theory of (left) skew monoidal categories and their variants (e.g. closed monoidal, symmetric monoidal), continuing the line of work initiated in recent years by Uustalu et al. [2, 3, 4, 5]. Skew monoidal categories are a weak version of Mac Lane's monoidal categories, where the structural laws λ, ρ, and α are not required to be invertible, they are merely natural transformations with a specific orientation. Uustalu et al. describe sequent calculi for numerous variants of skew monoidal categories, which can be identified as restricted substructural fragments of intuitionistic linear logic. These calculi enjoy cut elimination and admit a focusing strategy, sharing resemblance with Andreoli's normalization technique for linear logic [1]. The focusing procedure is useful for solving the coherence problem of the considered categories with skew structure.

Here we investigate possible extensions of the sequent calculi of Uustalu et al. with additive connectives. As a first step, we extend the sequent calculus in [4] with additive conjunction and disjunction ($\wedge$ and $\vee$), corresponding to studying the proof theory of skew monoidal categories with binary Cartesian products and coproducts. We introduce a new focused sequent calculus of derivations in normal form, which employs tag annotations in the style of [2] to reduce non-deterministic choices in bottom-up proof search. Apart from statements and proofs on pen and paper, we also want to formalize the focused sequent calculus and verify its correctness in the Agda proof assistant. We believe this to be beneficial for the development of modular normalization techniques for substructural logics arising as an extension of our sequent calculus, e.g. full Lambek calculus or intuitionistic linear logic.

This is work in progress in collaboration with Niccolò Veltri (Tallinn University of Technology).

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▸ ANDREAS WEIERMANN, *The phase transition for Harvey Friedman's Bolzano Weierstrass principle*.
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Let f be a weakly monotone and unbounded number-theoretic function. Harvey Friedman's Bolzano Weierstrass principle with respect to f is the following assertion (BW_f). $(\forall K \geq 3)(\exists M)(\forall x_1,\dots,x_M \in [0,1])(\exists k_1,\dots,k_K(k_1 < \dots < k_K \leq M \wedge (\forall l \leq K-2)|x_{k_{l+1}} - x_{k_{l+2}}| < \frac{1}{f(k_l)}))$. Friedman has shown that BW_f is true (by an application of the compactness of the Hilbert cube). Morever Friedman has shown that for $f(x) = 1/x^{1+\varepsilon}$ where $\varepsilon > 0$ the principle BW_f is not provable from $I\Sigma_1$. He also has shown that for $f(x) = \log(x)/x$ the assertion BW_f is provable from $I\Sigma_1$ and asked for the strength of BW_f for $f(x) = 1/x$ and $f(x) = 1/(x\log(x))$.

In our talk we answer these two questions and we give rather sharp bounds on the phase transition window for those functions f for which BW_f is provable or unprovable from $I\Sigma_1$. We also discuss the Friedman principle for monotone increasing sequences.

Finally as a real analysis spin off we obtain explicit formulas for the derivative of the smooth version of the inverse function of the inverses of the d-th branch of the Ackermann function for any natural number d.

▸ BARTOSZ WIĘCKOWSKI, *Towards a modal proof theory for reasoning from counterfactual assumptions.*

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In current research on structural proof theory, counterfactual inference is typically studied from a model-theoretic perspective. On this perspective, possible worlds models are methodologically basic. Model-theoretically defined consequence relations come first, and structural proof systems, usually transmitted via Hilbert-style axiom systems, have to be defined for these consequence relations. Structural proof theory is thus methodologically secondary. Labelled (or external) proof systems for counterfactual logics which incorporate possible worlds structures into their rules (e.g., [2, 3]) clearly illustrate this model-theoretic dependency. Importantly, the logics usually extend classical logic. By contrast, on the proof-theoretic perspective on counterfactual inference, we start from a certain primacy of inferential practice and proof theory. Proof-theoretic structure comes first. Meaning is explained in terms of proofs ([5]). Models are required neither for the formal explanation of the meaning of counterfactuals nor for that of counterfactual inference. Taking a proof-theoretic perspective and a constructive stance on meaning and truth (cf. BHK), we extend the rudimentary intutionistic subatomic natural deduction system for counterfactual implication presented in [8] with rules for conjunction. This proof system is *modal* insofar as derivations in it make use of *modes* of assumptions which are sensitive to the factuality status (factual, counterfactual, independent) of the formula that is to be assumed. This status is determined by means of a so-called reference proof system on top of which the modal proof system is defined. Specifically, the factuality status of atomic sentences is determined by the subatomic system (cf. [6]) of the reference system. The introduction and elimination rules for counterfactual implication and conjunction draw on this status. We establish normalization (cf. [4]) and the subexpression (hence, subformula) property for the system. On the basis of these results, we define a proof-theoretic semantics for counterfactual implication and conjunction, discuss the internal completeness (cf. [1]) of the system, and use the method of counter-derivations ([7]) to assess some familiar counterfactual fallacies and logics.

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▸ BOKAI YAO, *Forcing with urelements.*

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$\mathrm{ZFCU_R}$ is ZFC (with the Replacement Scheme) modified to allow a class of urelements. I first isolate a hierarchy of axioms based on $\mathrm{ZFCU_R}$ and then turn to forcing over countable transitive models of $\mathrm{ZFCU_R}$. A new definition of $\mathbb{P}$-names is given. The resulting forcing relation is full just in case the Collection Principle holds in the ground model. While forcing preserves $\mathrm{ZFCU_R}$ and many axioms in the hierarchy, it can also destroy the DC_{ω_1}-scheme and recover the Collection Principle. The ground model definability fails when the ground model contains a proper class of urelements.

▸ A.R. YESHKEYEV, I.O. TUNGUSHBAYEVA, G.YE. ZHUMABEKOVA, *The central type of a semantic pair.*

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We consider a hereditary [1] Jonsson theory T that is J-λ-stable [2]. Let C_T be a semantic model of T, and N, M be existentially closed submodels of C_T. A pair (N, M) is called existentially closed pair, if M is an existentially closed submodel of N. An existentially closed pair (C_T, M) is a semantic pair, if the following conditions hold: 1) M is $|T|^+_\exists$-saturated (it means that it is $|T|^+$-saturated restricted up to existential types); 2) for any tuple $\overline{a} \in C$ each its $\exists$-type in sense of T over $M \cup \{\overline{a}\}$ is satisfiable in C. We define the theory T' as follows: $T' = T \cup \{P, \subseteq\}$, where $\{P, \subseteq\}$ is an infinite set of existential sentences with constants from the existentially closed submodel in the considered existentially closed pair. Let T be a Jonsson L-theory and $f(\overline{x}, \overline{y})$ be an $\exists$-formula of L. If for any arbitrary large n there exists $\overline{a}_1, ..., \overline{a_n}$ in some existentially closed model of T, and $\overline{a}_1, ..., \overline{a_n}$ satisfies $\neg(\exists \overline{x}) \bigwedge_{k \leq n} f(\overline{x}, \overline{a_k})$, and for any $l \leq n$ $\neg(\exists \overline{x}) \bigwedge_{k \leq n, k \neq l} f(\overline{x}, \overline{a_k})$, then $f(\overline{x}, \overline{y})$ is said to have e.f.c.p. (existentially finite covered property). In the framework of the study of Jonsson theories, which are generally incomplete, and in some expanded language with new unary predicate and constant symbols, we refine in such generalization the earlier result obtained on beautiful pairs for complete theories from [3] (Theorem 6).

THEOREM 1. *Let T be a hereditary Jonsson $\exists$-complete theory. Then the following conditions are equivalent:*
1) T does not have e.f.c.p.;
2) two tuples $\overline{a}$ and $\overline{b}$ from the models of T' have the same type iff their central types [1] *in sense of T over M are equivalent by fundamental order;*
3) two tuples $\overline{a}$ and $\overline{b}$ from the models of T' and that are in M have the same type in sense of T' iff their central types are equal in sense of T.

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▶ AIBAT YESHKEYEV, OLGA ULBRIKHT, AND AIGUL ISSAYEVA, *Algebraically primeness and cosemanticness.*
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In [1] a criterion for the existence of a prime model for an arbitrary abelian group was found. The concept of an algebraically prime model generalizes the concept of a prime model.

DEFINITION 1. [2] A model A of a theory T is called an algebraically prime model of this theory if it can be isomorphically embedded in every model of T.

As shown in [3], no general criterion of algebraic primeness is known for an arbitrary theory. As is known from work [4], the theory of Abelian groups is a perfect Jonsson theory. The main result of this thesis is a criterion for the existence of an algebraically prime model for the theory of Abelian groups. In the work [5] gives criteria for the existence of different types of prime models and also for algebraically prime models for a particular case of Abelian groups, namely, for torsion-free Abelian groups. The results of works [1] and [5] are realized in the framework of the complete theories of the corresponding Abelian groups. Jonsson theory is, generally speaking, not complete.

The following theorem generalizes the main results from [1] and [5] on the language of cosemanticness ($\bowtie$), which generalized the notion of elementary equivalence.

THEOREM 2. *Let T be the theory of abelian groups. Then the theory T has an algebraically prime model if and only if at least one of the conditions is satisfied:*

a) $C_T \bowtie \oplus_p \mathbb{Z}_{p^\infty}^{(\alpha_p)}$*;*

b) $C_T \bowtie \oplus \mathbb{Q}^{(\beta)}$ *and* T^* *has an algebraically prime model,*
where C_T *is semantic model of Jonsson theory* T, $T^* = Th(C_T)$, $\alpha_p, \beta \in \omega^+$, $|C_T| = 2^\omega$.

All information about Jonsson theory and its details linked with a cosemanticness one can extract from [4].

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The discussions on truth, belief, disbelief, suspension, and their relationships to each other have been abundant in different philosophical fields, such as epistemology, truth theory, and logic. Our aim here is to discuss such notions within belief revision theory, and to point out some so far possibly unnoticed implications. We will focus on eight possible sentences about belief co., and specify our concern and separate the eight sentences into two groups, i.e. (Group1), consisting of beliefs/disbeliefs, and (Group2), consisting of non-beliefs/non-disbeliefs. Besides, we will formulate five assumptions, (A.1–A.5), and make some preliminary observation about these assumptions by means of a confrontation with [2] and [3]. Specifically, we will discuss some of our intuitive understandings of suspension, and formulate them in a group, i.e. (Group3), which consists of three kinds of belief suspension. Meanwhile, we will put forth two hypotheses, i.e. (H.S1–H.S2) about (Group3). Following, we will propose to re-frame them within the frameworks of ranking theory. In the last section, besides providing some final remarks, we will outline an open question about belief suspension.

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